Lecture 12: Quiz

Name:

Problem 1

The map \( T(x) = 2x^2 - 1 \) defines a chaotic dynamical system. For \( x = 0 \) for example, we get \( T(x) = 2 \cdot 0^2 - 1 = -1 \). The next number is \( T(T(x)) = 1 \). We get a sequence of numbers which are called an orbit. From the following 4 choices, two are an orbit. Which ones?

- a) 1, 2, 3, 4, ...
- b) 1, 1, 1, 1, 1, ...
- c) 2, 7, 97, ...
- d) 0, 0, 1, -1.

Problem 2

We have seen in lecture that if we iterate \( T(x) = 4x(1 - x) \) or \( S(x) = 4x - 4x^2 \) and start with \( x = 0.4 \), then we get different visible results after

- a) about 10 iterations
- b) about 100 iterations but not 10 iterations
- c) about 1000 iterations but not 100 iterations
- d) about 10000 iterations but not 1000 iterations

Problem 3

What is the Ulam-Collatz system?

- a) A differential equation showing chaotic behavior
- b) Take the sum of the denominators from a behavior
- c) Produce the pedal triangle number from a given triangle
- d) Divide by 2 if even and triple plus 1 if odd.

Problem 4

Which of the following dynamical systems is called the Lorentz system which produces the Lorenz attractor?

- a) \( \ddot{x} + x + (x^2 - 1)y = 0 \)
- b) \( \dot{x} = 10(y - x), \dot{y} = -xz + 28x - y, \dot{z} = \)
- c) \( x'(t) = x(t) \)
- d) \( x''(t) = -x(t) \)

Problem 5

Which of the following dynamical systems have a discrete time?

- a) The game of life
- b) Three body system
- c) The Kepler problem
- d) Ulam-Collatz system.

Problem 6

What is an example of a billiard dynamical system?

- a) The pedal map in triangles
- b) The game of life
- c) The Stadium
- d) The Collatz system.

Problem 7

Which dynamical system is used to find the roots of a function:

- a) The Feigenbaum map
- b) The Ulam map
- c) The Newton method
- d) The Kepler system

Problem 8

Which mathematician was the first to establish that low dimensional systems can exhibit chaotic behavior?

- a) Kepler
- b) Newton
- c) Poincaré
- d) Mandelbrot

Problem 9

Which mathematicians pointed out the concept of a strange attractor?

- a) Poincaré-Bendixon
- b) Hardy-Littlewood
- c) Ruelle-Takens
- d) Douady-Hubbard

Problem 10

Which movie features the "butterfly effect" and "strange attractors"?

- a) Jurassic park (1993)
- b) Butterfly dreaming (2008)
- c) Silence of the lambs (2001)
- d) Amelie (2001)