Lecture 12: Dynamical systems

Dynamics

Dynamical systems theory studies time evolution of systems. If time is continuous the evolution is defined by a differential equation \( \dot{x} = f(x) \). If time is discrete we look at the iteration of a map \( x \rightarrow T(x) \). The goal is to predict the future of the system when the present state is known.

Here is the prototype of a differential equation

\[
\begin{align*}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= rx - y - xz \\
\dot{z} &= xy - bz.
\end{align*}
\]

the Lorenz system. There are three parameters. For \( \sigma = 10, r = 28, b = 8/3 \), one observes a strange attractor.

Chaos

There are various definitions of chaos. One of them is "sensitive dependence on initial conditions". The smallest change of the initial state produce large changes in the future. We illustrate this with the example of the logistic, where already for \( n = 60 \), there is a big difference between the orbit of \( T(x) = 4x(1 - x) \) and \( S(x) = 4x - 4x^2 \).