The percentage of people using Apple OS X or the Gnu/Linux operating system is represented by a vector $\vec{v} = \begin{bmatrix} m \\ l \end{bmatrix}$. Let $\frac{2}{3}$ be the percentage of Mac OS X users, who switch to Linux each month and $\frac{1}{2}$ the percentage of Linux OS users, who switch to Apple each month. What is the distribution Linux/OSX users after 2 years if initially both groups have the same number of users.

The matrix $A = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix}$ has the property that $A \begin{bmatrix} m \\ l \end{bmatrix}$ is the distribution a month later. It is called a stochastic matrix: the sum in each column is 1. The dynamics $x(t) \mapsto Ax(t) = x(t + 1)$ is called a Markov chain.

AIM: COMPUTE $A^k \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$, the distribution after $k$ months for $k = 1$ and if you want for $k = 2$.

COMPUTE THE EIGENVALUES OF $A$.

COMPUTE THE EIGENVECTORS OF $A$. 
WHAT IS THE MATRIX $S$ for which $S^{-1}A S = B$ is diagonal.

WHAT IS $B^{24}$, up to 10 digits accuracy?

WHAT IS $A^{24}$?

WHAT IS THE DISTRIBUTION AFTER 24 YEARS?