HOMEWORK: 1.3:4,14,26*,34,48,50, Due: Tue 2/12/2002

COEFFICIENT MATRIX AND AUGMENTED MATRIX. Linear system
\[
\begin{align*}
3x - y - z &= 0 \\
-x + 2y - z &= 0 \\
-x - y + 3z &= 9
\end{align*}
\]
Rewrite it as \( A\vec{x} = \vec{b} \), with the coefficient matrix \( A \) and vectors \( \vec{b}, \vec{x} \).
\[
A = \begin{bmatrix}
3 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 3
\end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix}
\]
The augmented matrix is
\[
B = \begin{bmatrix}
3 & -1 & -1 & 0 \\
-1 & 2 & -1 & 0 \\
-1 & -1 & 3 & 9
\end{bmatrix}.
\]

GAUSS-JORDAN ELIMINATION. Gauss-Jordan elimination is an algorithm where successive subtraction of multiples of other rows or scaling brings the matrix into reduced row echelon form. The elimination process uses three possible steps, the elementary row operations:
- Swap two rows.
- Divide a row by a scalar
- Subtract a multiple of a row from an other row.
The process transfers a given matrix \( A \) into a new matrix \( \text{rref}(A) \) in reduced row echelon form.

REDUCED ECHELON FORM. A matrix is called in reduced row echelon form

1) If a row has nonzero entries, then the first nonzero entry is 1. (leading one)
2) If a column contains a leading 1, then the other column entries are 0.
3) If a row has a leading 1, then every row above has leading 1’s to the left.

RANK. The number of leading 1 in \( \text{rref}(A) \) is called the rank of \( A \).

SOLUTIONS OF LINEAR EQUATIONS. If \( A\vec{x} = \vec{b} \) is a linear system of equations with \( m \) equations and \( n \) unknowns, then \( A \) is a \( m \times n \) matrix.
The reduced echelon form of the augmented matrix \( B \) determines on how many solutions the linear system \( Ax = b \) has. We have the following three possibilities:
- **Exactly one solution.** There is a leading 1 in each row but not in the last row.
- **No solutions.** There is a leading 1 in the last row.
- **Infinitely many solutions.** There are rows without leading 1 and no leading 1 is in the last row.

If \( m < n \) (there are less equations then unknowns), then we have either zero or infinitely many solutions.
2) The vector

1) The entries

IN WORDS.

MATRIX "JARGON". A rectangular array of numbers is called a vector and matrix form of a linear equation.

EXAMPLES.

If the matrix has 3 rows

Matrices can be added, subtracted if they have the same size:

They can also be scaled by a scalar $\lambda$:

$\lambda A = \lambda$

$\lambda a_{11} \lambda a_{12} \cdots \lambda a_{1n}$

$\lambda a_{21} \lambda a_{22} \cdots \lambda a_{2n}$

$\cdots \cdots \cdots \cdots$

$\lambda a_{m1} \lambda a_{m2} \cdots \lambda a_{mn}$