FUNCTIONS OF ONE VARIABLES. A function of one variables \( f(x) \) assigns to a variables \( x \) a real number \( f(x) \). Example: \( f(x) = x \sin(x) \). We can visualize a function by its graph \( \{(x, y), y = f(x)\} \) which is a curve in space.

FUNCTIONS OF TWO VARIABLES. A function of two variables \( f(x, y) \) assigns to two variables \( x, y \) a real number \( f(x, y) \). Example: \( f(x, y) = \sin(xy) \), temperature distribution on a plate.

In the same way as functions of one variable were visualized by drawing the graph \( y = f(x) \) in space, we can visualize a function of three variables as a graph \( \{(x, y, z) | z = f(x, y)\} \) in space.

FUNCTIONS OF THREE VARIABLES. A function of three variables \( g(x, y, z) \) assigns to three variables \( x, y, z \) a real number \( g(x, y, z) \). Example: \( g(x, y, z) = \sin(xyz) \), temperature distribution in space. We can no more draw a graph of \( g \). But we can visualize it differently by drawing surfaces \( g(x, y, z) = c \), where \( g \) is constant.

SURFACES. Many surfaces can be described as a function of three variables \( g \). The points \((x, y, z)\) satisfying the equation \( g(x, y, z) = c \) for a surface. Examples.

**Graphs:** \( g(x, y, z) = z - f(x, y) \). If \( g(x, y, z) = 0 \), then \( z = f(x, y) \) and the surface is a graph of a function of two variables.

**Planes:** \( ax + by + cz = d \) is a plane orthogonal to the vector \( \vec{n} = (a, b, c) \). The equation says \( \vec{n} \cdot \vec{x} = d \). If a point \( \vec{x}_0 \) is on the plane, then \( \vec{n} \cdot \vec{x}_0 = d \). The equation can also be written as \( \vec{n} \cdot (\vec{x} - \vec{x}_0) = 0 \) which means that every vector \( \vec{x} - \vec{x}_0 \) is orthogonal to \( \vec{n} \).

**Quadrics** If \( f(x, y, z) = ax^2 + by^2 + cz^2 + dxy + eyz + fyz + gx + hy + kz + m \) the a surface \( f(x, y, z) = 0 \) is called a quadric. Below are some examples.
TRACES. To draw surfaces, it helps to look at the traces, the intersections of the surfaces with the coordinate planes \( x = 0, y = 0 \) or \( z = 0 \). Other points to look at are the intercepts, the intersections of the surface with the coordinate axes. The traces are shown in the pictures of the quadrics above.

For example: for the one-sided hyperboloid \( x^2 + y^2 - z^2 = 1 \) (called HYPERBOLOID I above), the \( z \)-trace is \( x^2 + y^2 = 1 \), a circle, the \( x \)-traces \( y^2 - z^2 = 1 \) is a hyperbola as well as the \( y \)-trace \( x^2 - z^2 = 1 \).