Motivate research

Teaching can motivate research. Even simple calculus or high school mathematics can be close to actual research mathematics. Here are some examples: while teaching calculus as a course assistant and learning ergodic theory, I wondered whether Green, Stokes and Gauss theorems can make sense in dynamics. They have and the corresponding de Rham cohomology is group cohomology. Indeed, calculus can be done without actually doing limits. An other example: a logician teaching single variable calculus can secretly investigate, which statements are actually provable with less axioms. A student interested in number theory can draw from varieties or elliptic curves to build examples in calculus. This can also enhance the pedagogy. Of course, these kind of thoughts must be shielded from the students but it can help to be more immersed into the problems and to stay excited about the subject even after teaching the same things over and over again.

Tapping research

Research can motivate teaching. Some problems in algebraic geometry for example might involve functions which lead to interesting calculus problems. It is amazing how much research has “shadows” which make sense in a calculus lecture. A number theorist for example can use the concept of linearization to solve Diophantine equations. There are many Diophantine equations which are unsolved and some of these problems are accessible for first year calculus students. Like for example the problem whether there are perfect Euler cubes, cubes with integer sides, for which all side diagonals and room diagonals have integer length.

Advertise the field

Research is a lot about politics and public relations, especially early in the career. Convincing the advisor, peers or friends that what you are doing is interesting and exciting. If research wants to be read, then you have to give talks which excite people. Producing clarity in a research seminar is similar to clarity in the classroom. And the track record of research talks is not so good from the didactic point of view. A good teacher is likely also able to advertise its own field.

Future grants

Students taught by you today might affect your research tomorrow. Not necessarily as mathematicians in your field, but as politicians who decide about NSF money. Doing a good job as a teacher is likely to sparks excitement in some of your students who will express this view at home, when talking to their friends etc. Your work in the classroom can have a snowball effect will likely affect you in the future. It certainly affects directly hundreds of people but indirectly thousands of people. Sharing teaching experiences on the web might reach even hundreds of thousands of people. Several thousand unique visitors visit a typical department server every day. Communicating mathematics well will assure that the subject will remain funded also, when you are a senior mathematician.