MATHEMATICA ROUTINES FOR THE DIRAC OPERATOR FOR
GRAPHS

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Abstract. Mathematica routines for the Dirac operator. This allows to com-
pute the cohomology for any graph. [1].

The first routine finds all the cliques in a graph.

\[ \text{Cli}[s_\ast, k_\ast] := \text{Module} \left\{ \{m, n, c, u, V, W, l = \{\}\} \right\}, \]
\[ V = \text{VertexList}[s]; \quad n = \text{Length}[V]; \]
\[ W = \text{EdgeList}[s]; \quad m = \text{Length}[W]; \]
\[ W = \text{Table}[\{W[[j, 1]], W[[j, 2]]\}, \{j, \text{Length}[W]\}] ; \]
\[ c = \text{Subsets}[V, \{k, k\}]; \]
\[ \text{If}[k == 1, l = V, \text{If}[k == 2, l = W, \]
\[ \text{Do}[ss = \text{Subgraph}[s, c[[j]]]; \]
\[ \text{If}[\text{Length}[\text{EdgeList}[ss]] == \text{Binomial}[k, 2], \]
\[ l = \text{Append}[l, \text{VertexList}[ss]], \]
\[ \{j, \text{Length}[c]\}]; 1]; \]

Now we compute the matrix \( D \).

\[ \text{Dirac}[s_\ast] := \text{Module} \left\{ \{q, d, b, u, m, n, p, v, DD\} \right\}, \]
\[ q = \text{VertexList}[s]; \quad n = \text{Length}[q]; \]
\[ d = \text{Table}[\{\{0\}\}, \{p, n - 1\}] ; \]
\[ l = \text{Table}[\{\}, \{p, n\}]; \quad b = \text{Table}[0, \{p, n\}]; \quad m = n; \]
\[ \text{Do}[\text{If}[m == n, \]
\[ \quad 1[[p]] = \text{Cli}[s, p]; \quad b[[p]] = \text{Length}[1[[p]]]; \]
\[ \quad \text{If}[b[[p]] == 0, m = p - 2], \{p, n\}]; \]
\[ v = \text{Sum}[b[[p]], \{p, n\}]; \]
\[ u = \text{Table}[\text{Sum}[b[[p]], \{p, 1, k\}], \{k, \text{Min}[n, m + 1]\}]; \]
\[ b = \text{Prepend}[b, 0]; \quad DD = \text{Table}[0, \{v, v\}]; \]
\[ \text{If}[m > 0, d[[1]] = \text{Table}[0, \{j, b[[2]]\}], \{i, b[[1]]\}]]; \]
\[ \text{Do}[d[[1, j, 1[[2, j, 1]]]] = -1, \{j, b[[2]]\}]; \]
\[ \text{Do}[d[[1, j, 1[[2, j, 2]]]] = 1, \{j, b[[2]]\}]; \]
\[ \text{Do}[\text{If}[m == p, \]
\[ \quad d[[p]] = \text{Table}[0, \{j, b[[p + 1]]\}], \{i, b[[p]]\}]; \]
\[ \quad \text{Do}[a = 1[[p + 1, i]]]; \]
\[ \quad \text{Do}[k = \text{Position}[1[[p]], \text{Delete}[a, j]]; \]
\[ \quad d[[p, i, k]] = (-1)^j, \{j, p + 1\}, \]
\[ \quad \{1, b[[p + 1]]\}], \{p, 2, n - 1\}]; \]

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This leads to the Laplace Beltrami operators which allow to compute the Betti numbers, and the Cohomology basis.

Example:

Do[
    If [m==p, Do[DD[[u[[p+1]]+j, u[[p]]+i]] = d[[p, j, i]],
        {i, b[[p]]}, {j, b[[p+1]]}], {p, 1, n-1};
        {DD + Transpose[DD], u}];
]

LaplaceBeltrami[s_] := Module[{DD, LL, br},
    {DD, br} = Dirac[s];
    LL = DD . DD;
    Table[Table[LL[[b[[k]]+i, br[[k]]+j]],
        {i, 0, Length[br]},
        {j, 0, Length[LL]}],
        {k, Length[br]}];
    Betti[s_] := Map[Nullity, LaplaceBeltrami[s]];
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\begin{verbatim}
DD = Transpose[DD];
\end{verbatim}

\begin{verbatim}
("finally build \hat{d}d^*"
\end{verbatim}

REFERENCES

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