The exponential map on a spheroid \( x^2/4 + y^2 + z^2 = 1 \). The primary caustic (the first root of the Jacobi field \( f \) starting at the initial point) is drawn in yellow, the secondary caustic (the second root of \( f(g(t)) \)) orange, the ternary red. One clearly sees the 4 cusps as the still unproven Jacobi's last geometric statement claims. 100 geodesics of the 6000 computed geodesics have been drawn. The picture was computed by solving the geodesic equations \( g^i\kappa = -G(i,j,k) g^i g^j \) (where \( G \) is the connection using Einstein notation) in conjunction with the Gauss-Jacobi equation \( f' = -K(g(t)) f \) (where \( K \) is the curvature of the surface) numerically with Mathematica. Since special needs are required (identifications of the map, assuring that we stay on the energy surface, checking whether the Jacobi field \( f \) reaches zero), the differential equations were "hand" integrated using Runge-Kutta and not using built-in DSolve routines. 6000 geodesics \( g(t) \) were computed on the ellipsoid and drawn in the spherical coordinate plane with \( (\theta, \phi) \) coordinate.
The non-rotationally symmetric ellipsoid

\[ \frac{x^2}{1.12} + \frac{y^2}{1.062} + z^2 = 1 \]

has caustics close to the antipode in the sphere case. We see again the primary, secondary and ternary caustic. 4000 geodesics
Geodesics on a non-rotationally symmetric ellipsoid