Common Sense and Mathematics: The Role of Quantitative Reasoning in Teacher Education and K – 12 Instruction

Maura Mast
University of Massachusetts Boston
maura.mast@umb.edu
Why quantitative reasoning?

• We are a world awash in numbers, a society “drenched in data”
• Quantitative reasoning/quantitative literacy: means more than being skilled at reading and writing – it also means being numerate.
• In a successful democracy, individuals must be able to think for themselves and make independent judgments.
• On a daily basis, we need to weigh information and make decisions.
Math Illiteracy Spells Trouble. Experts Say Americans' Struggle With Numbers Has High Cost: Wasted Money, Bad Policy, Unnecessary Risks
by Valerie Strauss, February 6, 2001       The Washington Post

Today's headlines reveal new dimensions of a problem that mathematicians deplore: "innumeracy," or math illiteracy, among many Americans who cannot tell millions from billions, average from median or percent from percentile.

The problem ... has important economic, social and personal consequences, mathematicians and others say. Money is wasted, bad policy is made, health and physical risks are taken based on faulty information.

"Surely demagogues yearn for people who can't recognize economic trade-offs, who lack a visceral feel for the difference between a million dollars for some cultural program, a billion dollars for some aircraft, or a trillion dollars in tax cuts," said Temple University mathematician John Allen Paulos.

The problem is not new. In 1981, David Stockton, budget director for President Ronald Reagan, laid bare the issue, saying, "None of us really understands what's going on with all these numbers."
“Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes...to approach complex problems with confidence in the value of carefully reasoning...QL empowers people...”

(from The Case for Quantitative Literacy)

“...if individuals lack the ability to think numerically they cannot participate fully in civic life, thereby bringing into question the very basis of government of, by, and for the people.”

(Robert Orrill, Preface)
Language is a flexible tool; mispronounce my name and I will still answer the telephone, although perhaps warily. But if you misdial my number I won’t have to bother deciding whether to answer or not.

In fact, plugging in wrong numbers can have far more serious consequences than merely annoying the public.

In 1999, the Mars Polar Orbiter burned up in the Martian atmosphere because engineers had miscalculated its orbit by using English units of feet and pounds instead of metric units....

This is not, by any stretch of the imagination, a numerically literate society, and yet there is a sense in which numbers have a certain dread authority. We are all the children of Pythagoras, after all, who said that nature is nothing but numbers.
Mathematics ≠ Quantitative Reasoning

• Mathematical literacy: often seen as basic arithmetic skills and algebra skills. Also seen as reasoning from the concrete to the abstract.

• Quantitative reasoning: elementary mathematics in difficult contexts
What mathematics is needed for quantitative reasoning?

- Arithmetic (including percentages)
- Estimation
- Visual representations of data
- Some probability and statistics
- Some modeling (linear, exponential)
- Some logic
- Some geometry
What else is needed for quantitative reasoning?

• Reading – and critical reading
• Writing – summary and analysis
• Estimation
• Paying attention to the numbers
• Common sense
• Common knowledge
• Habits of mind
How has the mathematics curriculum in the K – 16 arena changed?

• Earlier: utilitarian focus, emphasis on arithmetic skills
• Algebra was for the college prep track
• After WWII: push to increase science and math education, calculus moved to 1st year of college
• “new math” – made people wary of change
• Now: calculus is at the junior year of high school, pre-algebra in elementary; algebra in 7th grade (“algebra as the gatekeeper”)
• At the same time, technology is here to stay: Wolfram Alpha, Google calculator, virtual manipulatives, etc.
• And: greater expectations (from the workplace) for quantitative knowledge and abilities.
• With a curriculum that advocates “Algebra II for all” we push students to master specific topics with the argument that the payoff will come later – when they take higher level mathematics courses.

• The reality is:
  – High levels of remediation (some estimates put this at one in three students)
  – Lack of understanding of basic statistics and data
  – High failure rates – and repeat rates – for mathematics classes in high school
I'll never get this second problem.

Just put down "eleven," Franklin, and don't worry about it... that's what I did.

"X" is almost always eleven, and "y" is almost always nine...

One thing I've learned about algebra... don't take it too seriously...
Common Core Standards for Mathematical Practice

• Make sense of problems and persevere in solving them.
• Reason abstractly and quantitatively.
• Construct viable arguments and critique the reasoning of others.
• Model with mathematics.
• Use appropriate tools strategically.
• Attend to precision.
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.
- This curriculum reform is more than a reorganization or shuffling of content.
- It acknowledges the importance of mathematical habits of mind.
- This means
  - Conjecturing
  - Inventing
  - Reasoning from patterns
  - Describing
  - Experimenting
  - Considering multiple points of view
  - Reasoning
- “There is another way to think about it, and it involves turning the priorities around. Much more important than specific mathematical results are the habits of mind used by the people who create these results.” (Cuoco, Goldenberg, Mark, http://www2.edc.org/CME/showcase/HabitsOfMind.pdf)
By emphasizing the teaching of quantitative reasoning, we can give K – 12 students the opportunity to deepen their understanding of mathematical concepts. Why? QR cannot be taught with the standard “explanation – example – exercises” approach. Every QR problem is different and it is the habits of mind that are the key here.

The potential: “A significant amount of work on QR can actually reduce the overcrowding [in the curriculum] by reducing the large amount of time (up to 35%) spent re-teaching concepts and skills (an ineffective approach to remedying misconceptions).” (Burkhardt, Calculation vs. Context: QL and its Implications for Teacher Education, 2008)
• If QR is only in a “math” course, then students will think it only applies to “math” situations – which many of them will avoid as much as possible.

• Transferability is always an issue: this argues for QR across the curriculum.

• It’s also important to acknowledge the role of QR in making and supporting arguments. This is another cross-curricular way to teach QR.
What does this tell us about bringing quantitative reasoning into teacher preparation?

• The Common Core standards give a good opportunity, because they explicitly address the importance of habits of mind.

• The standards call for eight standards for mathematical practice that incorporate NCTM process standards along with strands of mathematical proficiency such as adaptive reasoning and conceptual understanding.

• In particular: “productive disposition” described as “habitual inclination to see mathematics and sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.”
Challenges

• Teacher education
• State standards and high stakes testing
• Uneven professional development
• Textbook industry
• Curriculum directors: local, regional, state
• School boards
• Math wars
AAAS: Science for all Americans (1990)

• It is quixotic to believe that elementary and secondary education in American – serving nearly 50 million students located in more than 80,000 schools and 50 states – could easily or quickly be changed. Even with great ideas, the best of intentions, and investment of resources on a scale appropriate to the job, and lots of hard work, any sweeping change in the educational system nationally is bound to take a decade or longer.

• It is more than simply a problem of scale, however. ...the system of education in the United States is decentralized politically and economically. Decisions on educational policy and the use of resources for education are made by literally thousands of different entities...

• Ultimately, reform is more about people than it is about policies, institutions, and processes.... Professions may change mostly in response to turnover.... Reforming teachers’ education, therefore, is the *sine qua non* of school reform, but it will necessarily be slow to make its impact felt.
Significant challenges, but significant opportunities

• “The mathematical preparation of teachers is both a core problem in and a central solution to improving K – 12 mathematics education nationwide.” (Heaton and Lewis, *Notice of the AMS*, March 2011)

• Their approach: a partnership between mathematicians and mathematics educators that builds on the CBMS recommendation
  
  Prospective teachers need a solid understanding of mathematics so that they can teach it as a coherent, reasoned activity and communicate its elegance and power. Mathematicians are particularly qualified to teach mathematics in the connected, sense-making way that teachers need. For maximum effectiveness, the design of this instruction requires collaboration between mathematicians and mathematics educators and close connections with classroom practice.

• This includes a course called Contemporary Mathematics: “...which introduces students to many ways in which mathematics is important to our daily lives.” This is part of the 12 hours that elementary education majors take for their degree.
Some strategies to bring quantitative reasoning into teacher education

• Quantitative reasoning is now an accepted part of general education. We are no longer arguing or discussing about the validity of teaching quantitative reasoning at the college level – or what quantitative reasoning is – the conversation has now shifted to a discussion of how to teach it.

• Implications: it’s likely that elementary education students will be exposed to quantitative reasoning through a specific course or through QR across the curriculum.

• Can we build on that? Can we make an explicit link for these students?

• What about secondary education teachers? They are more likely to have a mathematics major. How do they get QR?
Make QR an integrated part of teacher education

• This is a daunting task (involving licensing, accreditation, teacher education programs) but we have leverage with the Common Core call for quantitative reasoning and habits of mind.
• We have some successes to reflect on, e.g. the movement to make statistical literacy a real part of the K–12 curriculum.
• “Lasting change begins with a clear conception of the measurable features of numeracy, the establishment of a course of study..., the specifications of new requirements for the teaching license, the redesign of license tests, recognition in the accreditation and state approval standards, and incorporation in the state’s curriculum assessments.” (F. Murray, Calculation vs. Context: QL and its implications for Teacher Education, 2008)
• It also gives us new opportunities to incorporate new pedagogies.
Other approaches

• Curricular change bringing explicit attention to QR in elementary education programs
• Professional development programs
• Course development at the secondary level emphasizing QR (which brings about professional development); e.g., *Advanced Mathematical Decision Making* and other alternative fourth-year high school courses.
• Carnegie Foundation Statway/Quantway work: focused at the community college level but could be brought to the secondary classroom.
• Panels, talks, presentations at local, regional, state, national meetings (e.g. NHATME meeting Nov. 2010 was focused on QL)

• Participation in state-wide discussions of teacher preparation, implementation of standards, etc.

• Learning communities for teachers

• Math teacher circles
Derek Bok:
Like learning to write well or speaking a foreign language, numeracy is not something mastered in a single course. The ability to apply quantitative methods to real-world problems requires a facility and an insight and intuition that can only be developed through repeated practice. Thus quantitative material needs to permeate the curriculum, not only in the sciences but also in the social sciences and, in appropriate cases, in the humanities, so that students have opportunities to practice their skills and see how useful they can be in understanding a wide range of problems.

*Our Underachieving Colleges, 2006*
Can we do it?

• AAAS: “reform requires creating conditions for change... at the same time as barriers to reform are being removed, positive conditions for change must be established. They need to emphasize creating an environment for teachers and administrators that encourages experimentation, focus on long-term gains rather than on such immediate goals as raising test scores, and recognize and reward innovation”

• We have an opportunity with the Common Core standards to re-focus teacher education and professional development to emphasize quantitative reasoning and the habits of mind.