The Common Core State Standards
What the Standards for Mathematical Practice Can Mean for Teachers and Mathematicians

Juliana Belding and Ginger Warfield
Harvard University and University of Washington

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An Opening Problem: Treasure Island

You and two friends, Alice and Bob, are on an island and you each independently find directions to a buried treasure.

*The two largest palm trees on the island serve as markers for the treasure. Count your paces from your location to the western-most palm tree. Once you arrive, make a right-angle turn to the right and walk the same number of paces. Mark this spot. Go back to your original location and count your paces to the eastern-most palm tree. Then make a right-angle turn to the left and walk the same number of paces and mark this spot. The treasure is located directly in the middle of the two spots you’ve marked.*

Which of you finds the treasure?
The Standards for Mathematical Practice

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
What are the Common Core State Standards?

- State-wide effort led by National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO)
- Purpose: “To provide a clear and consistent framework to prepare our children for college and the workforce.”
- Released in June 2010, now adopted formally by 44 states and DC
The Standards for Mathematical Practice

“varieties of expertise that mathematics educators at all levels should seek to develop in their students”

These have historical roots in the “processes and proficiencies” of, for eg.,

- NCTM Process Standards, 2000
- National Research Council’s Adding It Up Report, 2001
What do these Mathematical Practices mean... to Teachers?

Are they Obvious?
- Aren’t these just the usual “problem-solving strategies”?

Are they Ambiguous?
- Does #5 mean “tools” like protractors and calculators?
- Does “precision” in #6 mean “enough decimal places”?
- What’s the difference between #7 and #8?
  - #7: Look for and make use of structure.
  - #8: Look for and express regularity in repeated reasoning

And just what do they look like in practice?
What do Mathematical Practices mean...

to Mathematicians?

- Un-codified, highly individual, not limited to a single list
- Usually not explicitly taught (no ‘Math Research Methods’ course)
- Second nature or learned habits of mind?

*How* did those of us trained as mathematicians learn ‘mathematical practices’?
Apprenticing Mathematical Practices

- In well-taught lectures and classes?
- Through ‘reading’ texts and papers on our own?
- Through research in undergrad, grad and postgrad?
- Through hands-on learning of unfamiliar mathematics?
- Through collaboration with advisors, peers and colleagues?

How can this inform teacher professional development around mathematical practices?
Designing Professional Development around Mathematical Practices

- Focused on mathematical content
  (not pedagogy, initially)

- Extended investigation into familiar and unfamiliar math
  ("low threshold, high ceiling")

- Explicit Reflection
  - Mathematical (on content and practices)
  - Pedagogical (how to translate to classroom)
Why should Mathematicians be involved?

- Our experience with ‘mathematical practices’ in practice
- Our viewpoint of unifying concepts in math (bridging, connecting)
- Our role in K-16 spectrum as college and university instructors
An Example: Focus on Math, Boston Area

Focus on Math

“a unique partnership ... to improve student achievement by providing mathematics teachers with the content knowledge and skills valuable in their profession.”

- Greater Boston School Districts (7 Districts)
- Boston University, UMass Lowell, Lesley University
- Educational Development Center, Inc. (EDC)
- (an NSF-funded MSP)
The Audience: Middle and High-School Teachers (5 MS/12 HS)

The Math: Geometric Transformations and Arithmetic 
from plane w/o coordinates to plane w/ coordinates to complex numbers

The Goals:
- Vertical content connections related to geometry, congruence, similarity, functions and complex number arithmetic
- Exploration of the Standards for Mathematical Practice
The Origin Developed in Spring 2011 by group of
  ➤ MS/HS ‘master teachers’ (7)
  ➤ math educators/mathematicians (4)
Co-led by two HS teachers in August 2011

The Overview 4 days, each with
  ➤ Problem Sets (series of scaffolded questions)
  ➤ Geometer’s Sketchpad Lab Exploration
  ➤ Summary of Mathematics from Day
  ➤ Reflection on “problem-solving strategies/mathematical habits"
The two largest palm trees on the island serve as the markers for the treasure. You must first start at the secret location. Count your paces to the western most palm tree. Once you arrive at the tree take a sharp right turn and walk the same number of paces. Mark this spot. Return to the secret location and walk to the eastern most palm tree, again counting your paces. Take a sharp left turn and walk the same number of paces and the mark this spot. The treasure is located directly in the middle of the two spots that you have marked. Good luck.

Does the location of the treasure depend on the secret location?
Articulating Mathematical Practices: Day 1 and 2

Day 1:
- Working together/Mathematical discussion
  ‘Draw, graph and talk’
- Trial and error
- From concrete to abstract
  ‘Start w concrete (start with numbers and then generalize with variables)’

Day 2: Very problem specific, for eg:
‘translate to origin, rotate and translate back’
More generalization of the specific strategies, including:

- **Experiment First**: 'simplify problems by adding constraints’, ’use specific numbers then generalize’, ’chunking, breaking down into manageable parts’
- **Collaboration helps** ’divide and conquer’, ’keep us from quitting’, ’[your work] reminds someone else of something’
- **Learn from prior knowledge** ’firm up understanding of each step before going onto new step, stop and summarize’
- **Use alternate representations** ’look at geometric/alg representation’
Our Strategies vs. the Standards for Mathematical Practice

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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5. Use appropriate tools strategically.
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“What examples of these practices have we seen in action?”
Back to the Classroom: “The Kids"

In three groups (MS, HS Geometry, Advanced HS):

How would you **scaffold the opening problem** so your students could develop the tools to solve it?

*Which content standards would arise in this context?*

*Which practice standards would arise in this context and why?*
Assessing the Common Core State Standards

Two multi-state US Dept of Education Funded Assessment Groups:

- Partnership for Assessment of Readiness for College and Careers (PARCC)
  http://www.parcconline.org/about-parcc

- SMARTER Balanced Assessment Consortium (SBAC)
  http://www.k12.wa.us/smarter/

This is happening now through 2014...
Resources and Questions?

- Common Core State Standards
  www.corestandards.org
- The ‘Focus on Math’ Math Science Partnership
  www.focusonmath.org
- “Habits of Mind”, Cuoco, Goldenberg, Mark, 1996