

MATH 233A (2009), PROGRAM

Week 1.

Day 1: No class

Day 2: Sheaves and presheaves, kernels, problem for cokernels.

Week 2.

Day 1: Thm about associated sheaf (3/4 through the proof). Cor.: sheaves are an abelian category.

Day 2: Proof of the associated sheaf construction, direct and inverse images, Cech complex for a cover.

Week 3.

Day 1: Recap Zariski topology, construction of the the structure sheaf, construction of sheaves associated to modules as an adjoint functor. Notion of quasi-coherent sheaf on $Spec(A)$. Notion of ringed and locally ringed spaces. Proof that $Spec$ is functor $Rings \rightarrow \mathcal{L}ocally\ Ringed\ Spaces$.

Day 2: Definition of differentiable manifolds as locally ringed spaces, proof that $Hom_{Rings}(A, B)$ is the same as map between spectra as locally ringed spaces. Definition of schemes. Open subscheme of an affine is a scheme. Map to an affine via functions. Example of punctured plane.

Week 4.

Day 1: Quasi-coherent sheaves (inverse and exact images), closed embeddings, local properties (reducedness, Noetherianness, etc.)

Day 2: Away from class.

Week 6.

Day 1: Yoneda functor (S -points), Zariski sheaves, representable morphisms, criterion for representability of a functor, relative $Spec$, attempt to define the projective space as a functor.

Day 2: Problem solving sessions—vector bundles.

Week 7.

Day 1: Projective space, proof of existence as a scheme, the line bundles, coordinates, calculation of sections of $O(n)$, defn. of projective and quasi-projective varieties, introduction to Serre's theorem about graded modules.

Day 2: Recap projective space, relative version, Grassmannians, Segre, Veronese and Plucker embeddings.

Week 8.

Day 1: equations for Segre, Veronese and Plucker. Statement of Serre's theorem (graded modules over Sym vs. $QCoh$ on the projective space). Corollaries: generation by global sections. Serre subcategories and quotients (beginning).

Day 2: Serre quotient (end). Blow up of the vector space at 0. Interpretation of the functor $\{QCoh(P(V)) - Graded\ Sym\text{-modules}\}$ via $V-0$. Algebraic groups, actions, equivariant q.c. sheaves (beginning).

Date: November 12, 2009.

Week 9.

Day 1: Discussion of HW problem: affine schemes characterized by the exactness of global sections functor. Equivariant q.c. sheaves; representations, equivariant with respect to \mathbb{G}_m . End of proof of Serre's theorem.

Day 2: Equivariance via S -points. Projective space as a quotient: idea of sheafifying the quotient presheaf. Statement of Serre's theorem (finite-dimensionality of sections of coh. sheaves on the projective space). Attempt to prove Chevalley's theorem (reassigned as HW). Separated schemes. Čech cohomology (beginning).

Week 10.

Day 1: Proof of finite-dimensionality theorem of cohomologies of coherent sheaves on the projective space.

Day 2: $\text{Proj}(A)$. $\text{Proj}(A)$ is projective.

Week 11.

Day 1: Proper maps. $\mathbb{P}(V)$ is proper.

Day 2: Valuative criteria. Distribution of projects.

Week 12.

Discussions of projects.