

Comments on Problem Set 6

and a little more on PS5

Math 250a

October 29, 2001

Many of these homework problems are important facts in their own right, so if you didn't get something right on the problem set, be sure to track it down in the solutions below. The books cited here are available in Birkhoff.

Problem 2. [Se, p. 117]. Page 118 also contains token remarks on why Res maps into $H^1(H, A)^{G/H}$.

Problem 4. [Se, p. 125].

Problem 6. ii) Both H^1 groups are actually zero.

Let $M =$ (the $ax + b$ group over K). The following exact sequence was exhibited in class:

$$1 \longrightarrow \mathbf{G}_a(K) \longrightarrow M \longrightarrow \mathbf{G}_m(K) \longrightarrow 1$$

where $\mathbf{G}_a(K)$ is the additive group of K and $\mathbf{G}_m(K)$ is the multiplicative group. Problem 4 then yields a long exact sequence of cohomology groups, a portion of which is

$$H^1(K/k, \mathbf{G}_a(K)) \longrightarrow H^1(K/k, M) \longrightarrow H^1(K/k, \mathbf{G}_m(K))$$

It was proved in class that both $H^1(K/k, \mathbf{G}_a(K))$ and $H^1(K/k, \mathbf{G}_m(K))$ are zero, so $H^1(K/k, M) = 0$.

The case $M = \mathrm{SL}_n(K)$ is similar. There is an exact sequence

$$1 \longrightarrow \mathrm{SL}_n(K) \longrightarrow \mathrm{GL}_n(K) \xrightarrow{\det} K^* \longrightarrow 1$$

and a portion of the long exact sequence of cohomology groups is

$$\mathrm{GL}_n(k) \xrightarrow{\det} k^* \longrightarrow H^1(K/k, \mathrm{SL}_n(K)) \longrightarrow H^1(K/k, \mathrm{GL}_n(K))$$

The determinant map is surjective and $H^1(K/k, \mathrm{GL}_n(K)) = 0$, so $H^1(K/k, \mathrm{SL}_n(K)) = 0$.

More on Problem Set 5

Problem 6 of set 5 is covered in [Se, p. 154].

For Problem 7, see [We, p. 182].

References

[Se] J.P. Serre, *Local Fields*, Springer, 1979.

[We] C. Weibel, *An introduction to Homological Algebra*, Cambridge Univ. Press, 1994.