

MATHEMATICS 131: TOPOLOGY I

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Welcome. This course is intended for undergraduates with a concentration in mathematics or a closely related discipline.

Topology is a cornerstone for many areas of serious mathematical research. This includes all kinds of analysis, differential geometry, algebraic topology, and algebraic geometry. The set of topological facts you will need later in your mathematical career vary wildly with the chosen focus, however, and one of the principal aims of this course is to equip you with the tools needed to use topology in any of these arenas.

The formal prerequisites are “some acquaintance with metric space topology (Mathematics 23a,b, 25a,b, 55a,b, 101, or 112) and with groups (Mathematics 101, 122 or 55a).”

In addition, students will be expected to be familiar with reading and writing proofs. I emphasize that a relatively high level of mathematical sophistication is expected in this course; as a result, the course will not be particularly leisurely in pace or in content.

Objective. Your objective in this course is to learn the basics of topology. At minimum, this includes the development of a good understanding of the following:

- the category of topological spaces — topological spaces, continuous maps, homeomorphisms, products, fibers, pushouts, quotients, and function spaces;
- basic properties of topological spaces — compactness, connectedness, countability, paracompactness, sobriety, and the separation axioms;
- key theorems of topology — Urysohn’s lemma, the Tietze extension theorem, and the Nagata-Smirnov metrization theorem; and
- the basics of homotopy theory — homotopies, fundamental groups, and covering spaces.

A successful student in this course will leave with a solid understanding of these topics.

Textbook. The textbook for this course is:

Munkres, James R. *Topology*. Prentice Hall; 2nd ed. (2000). xvvi+537 pp.
Hardcover. ISBN-13: 978-0-1318-1629-9.

This book is available at the Coop. It also seems to be available from several online vendors. The first half of the course will use the text heavily; the second half will rely on it less. Over the course of the semester, you will be expected to read most of it.

Office hours. I will be available in my office on Mondays and Wednesdays, immediately before this course, from 11 AM until 12 PM. You may also email me to make an appointment; however, please contact me at least 24 hours before you wish to meet.

Course assistant. Our course assistants are LAURA STARKSTON and ERNEST FONTES. We will establish times for a recitation section once the attendance in the course has stabilized. In the meantime, they are available to answer questions via email. Please give them 24 hours to answer your questions during the week, and as much time as they need on holidays and weekends.

Evaluation. My operating assumption will be that you are motivated, intelligent adults. You recognize, I assume, that developing a real understanding of the material is its own reward. So I feel no particular need to bribe you with grades in order to get you to learn the material, and I have no interest in playing some perverse game in which I hold your futures hostage as a means of coercing you to complete the coursework.

There is nevertheless a need to evaluate how successfully you have acquired the skills and knowledge this course is meant to impart, both so that I know I am conveying the material well and so that you know you are mastering what you'll need for your future work in mathematics. To that end, I will ask you to complete a large number of homework exercises, which will be the sole basis on which your grade will be computed.

Homework. I will assign exercises during every class, which should be turned in *one week after they have been assigned*. In these exercises, I will ask you to prove a number of the basic results needed to follow the content of the course.

There will be a quite a lot of homework in the course, and many of the exercises will be very difficult. On average, you may expect to be assigned six exercises in each class, of the following two types. Five of these will be “boilerplate,” meaning that you will not have to have a significantly new idea to perform the exercise; these exercise are intended to demonstrate the utility of the concepts and theorems we have covered. (Note that “boilerplate” does *not* mean “trivial.”) One of these exercises will be “starred,” meaning that a significantly novel idea will be required to complete the exercise.

The exercises will be sequentially numbered; in all you may expect to be assigned around 120 exercises. You are asked to complete each of them.

You are encouraged to work in groups, but there are three guidelines you should follow:

- *Write the solutions yourself.* I will not accept papers with multiple authors, as I need to be able to evaluate your proof-writing abilities individually.
- *Give credit where credit is due.* If the key idea for a proof or a computation is someone else's, indicate this clearly. For instance, you may write, “This argument is due to G. Jetson.” or “This was explained to me by W. Flintstone.” It is common in mathematics research to borrow ideas, but it is important that they be correctly attributed; otherwise this practice is indistinguishable from *charlatanism*.
- *Contribute.* Please do not take advantage of your intelligent but meek friends' tolerance. Be sure that you are making a genuine contribution to the group's work. If you are not, please find another group. Sponges cannot learn as much material as those who put forward consistent effort.

In-class presentations. Beginning in Week 3, every Friday, I will ask some of you to present your solution to a recent homework problem. (Eventually everyone in the class will present a number of times.) Your colleagues and I will also ask you various questions during your presentation. The purpose of this is twofold: first, you will learn to present mathematical ideas effectively and comfortably in a public environment, and second, you will have the opportunity to hear other perspectives and approaches toward the material.

Exam. There are no in-class exams for this course. In lieu of a final exam, there will be a long, challenging (but hopefully fun and intriguing) homework assignment at the end of the semester.

Policy. Again, my operating assumption will be that I am working with motivated, intelligent adults who will treat one another with respect and courtesy. This plays out in a variety of ways:

- The difficulty of the material, combined with the differences in skills of your colleagues, can engender plenty of frustration. To some extent, this is to be expected. Do not permit that frustration to get in the way of your education, or the education of your colleagues.
- It is impossible to impart the entirety of the knowledge you are expected to acquire in this course during lectures. You should expect the bulk of your learning to result from the reading and from the exercises you will perform.
- Please do not hesitate to provide me with appropriate, timely feedback. My lectures are interactive, and my pace is determined in part by the responses I get from students. If the course is moving at a pace or in a direction with which you are uncomfortable, please do not wait until your head explodes (either from confusion or from boredom) before speaking with me.

Plan for the semester. This is here only to give an overview; there may be cause to go faster or slower through the material. I estimate the margin of error for this list of topics to be about two weeks.

Week 1 (2, 4 Sep.) The category of topological spaces I: open sets, closed sets, closure interior, density. (Chap. 1; Sect. 12, 17, 18)

Week 2 (9, 11 Sep.) The category of topological spaces II: continuous maps and homeomorphisms. (Chap. 1; Sect. 12, 17, 18)

Week 3 (14, 16, 18 Sep.) Constructing topologies: bases and subbases, subspaces, quotients, pullbacks, pushouts, metric spaces. (Chap. 2)

Week 4 (21, 23 Sep.) Compactness I: compact spaces, Tychonoff's theorem, compact metric spaces, compact-open topology. (§§ 26, 27, 37, 43, 45)

Week 5 (28, 30 Sep., 2 Oct.) Compactness II: compactification, limit point compactness, local compactness, paracompactness, partitions of unity. (§§ 38, 28, 29, 39, 41)

Week 6 (5, 7, 9 Oct.) Countability and separation: first and second countable spaces, the T_i conditions, Urysohn's lemma, Tietze extension theorem. (§§ 30, 31, 32, 33, 35)

Week 7 (14, 16 Oct.) Metrization theorems: Urysohn's metrization theorem, the Nagata-Smirnov metrization theorem, the Smirnov metrization theorem, Ascoli's theorem. (§§ 34, 40, 42)

Week 8 (19, 21, 23 Oct.) Sheaf theory I: presheaves, morphisms, stalks.

Week 9 (26, 28, 30 Oct.) Sheaf theory II: sheaves, constant sheaves, locally constant sheaves, and local homeomorphisms.

Week 10 (2, 4, 6 Nov.) Sheaf theory III: *espace étalé*, sheafification.

Week 11 (9, 13 Nov.) Covering space theory: covering spaces, universal covering spaces, the étale fundamental group.

Week 12 (16, 18, 20 Nov.) Computing the étale fundamental group: Seifert-van Kampen theorem, simple connectedness.

Week 13 (23, 25 Nov.) Homotopy theory: Paths and π_0 , homotopies of maps, homotopy invariance of the étale fundamental group.

Week 14 (30 Nov., 2 Dec.) The comparison theorem.