For each of the equation below, solve for $x$.

1. Warm up.
   (a) $7^x = 12$  
   (b) $x^7 = 12$  
   (c) $\log_7 x = 12$

2. These involve a single base.
   (a) $7^{4x+3} = 12$  
   (b) $5 \cdot 7^{x+3} = 7^{2x+1}$  
   (c) $7^x - \frac{49}{7^{2x}} = 0$

3. These involve multiple bases. You do not need to simplify the answers as they are ugly.
   (a) $4^{x+3} = 5^{2x-1}$  
   (b) $7 \cdot 4^{x+2} = \frac{5}{3^{2x+1}}$  
   (c) $2^{x^2} = 5 \cdot 3^x$

4. These are easiest with a substitution.
   (a) $e^x(e^x - 5) = 6$  
   (b) $2e^{2x} + 6 = 7e^x$  
   (c) $7^x - \frac{8}{7^x} = 7$
5. More practice.
   (a) \(2^{2x+3} = 8^{x-7}\)  
   (b) \(3^x \cdot \frac{5}{3^x+1} = 0\)  
   (c) \(C^x = 3^{x+7}\)  

   (d) \(B^{2x} = 2^{Bx}\)  
   (e) \(3^{2x-1} - 3^x = \frac{10}{3}\)  
   (f) \(4^{\log_2 x} = 7\)
Solving Equations with Exponentials and Logarithms — Solutions

1. (a) \( x = \log_7 12 \)
(b) \( x = \sqrt[12]{12} \)
(c) \( x = 7^{12} \)

2. (a) Taking \( \log_7 \) for both sides, we have \( 4x + 3 = \log_7 12 \). Hence
\[
x = \frac{1}{4}(\log_7 12 - 3)
\]
(b) Putting the base 7 terms on the same side, we have \( 5 = 7^{x^2} \). Hence
\[
x = \log_7 5 + 2
\]
(c) After expressing everything in base 7, we have \( 7^x = 7^{2-x^2} \), so the exponents must be equal: \( x = 2 - x^2 \). Solving \( 0 = x^2 + x - 2 = (x + 2)(x - 1) \), we have \( x = -2, 1 \).

3. (a) Take \( \ln \) for both sides. Then \( (x + 3) \cdot \ln 4 = (2x - 1) \cdot \ln 5 \). Putting the terms involving \( x \) on one side, we have \( (\ln 4 - 2 \ln 5)x = -3 \ln 4 - \ln 5 \). Hence
\[
x = \frac{-3 \ln 4 - \ln 5}{\ln 4 - 2 \ln 5}
\]
(b) We do the same thing as the previous problem.
\[
\ln 7 + (x + 2) \cdot \ln 4 = \ln 5 - (2x + 1) \cdot \ln 3
\]
\[
(\ln 4 + 2 \ln 3)x = \ln 5 - \ln 3 - \ln 7 - 2 \ln 4 = \ln 5 - \ln 336
\]
\[
x = \frac{\ln 5 - \ln 336}{\ln 4 + 2 \ln 3}
\]
(c) Taking \( \log_2 \) for both sides, we have \( x^2 = \log_2 5 + x \cdot \log_2 3 \). Hence we have the quadratic equation
\[
x^2 - \log_2 3 \cdot x - \log_2 5 = 0
\]
The quadratic formula gives
\[
x = \frac{\log_2 3 \pm \sqrt{(\log_2 3)^2 + 4 \log_2 5}}{2}
\]

4. (a) We let \( y = e^x \). The equation becomes \( y(y - 5) = 6 \), or \( y^2 - 5y - 6 = 0 \). Hence we have \( (y - 6)(y + 1) = 0 \), namely \( y = e^x = 6, -1 \). As exponential function is always positive, we only have one solution \( x = \ln 6 \).
(b) Again we let \( y = e^x \), and the equation becomes \( 2y^2 - 7y + 6 = 0 \), or \( (2y - 3)(y - 2) = 0 \). Hence \( y = e^x = \frac{3}{2}, 2 \). We have \( x = \ln \frac{3}{2} \) or \( \ln 2 \).
(c) We let \( y = 7^x \), and the equation becomes \( y^2 - 7x - 8 = 0 \), or \( (y - 8)(y + 1) = 0 \). Hence \( y = 7^x = 8, -1 \), or \( x = \log_7 8 \).
5. (a) Writing everything in base 2, we have \(2^{2x+3} = 2^{3x-21}\). Equating the exponent gives us \(2x + 3 = 3x - 21\), or \(x = 24\).

(b) The equation simplifies to \(\frac{5}{3} = 0\), which is impossible.

(c) Taking \(\ln\) for both sides, we have
\[
x \cdot \ln C = (x + 7) \cdot \ln 3
\]
\[
x \cdot (\ln C - \ln 3) = 7 \ln 3
\]
\[
x = \frac{7 \ln 3}{\ln C - \ln 3}
\]

(d) This equation is always true if \(B^2 = 2^B\), and always false if \(B^2 \neq 2^B\). Hence in the former case any number is a solution, and in the latter case there is no solution.

(e) Let \(y = 3^x\). Then the equation is \(\frac{y^2}{3} - y = \frac{103}{y} = 0\), or \(y^2 - 3y - 10 = (y - 5)(y + 2)\). Hence \(y = 3^x = 5, -2\), or \(x = \log_3 5\).

(f) The left hand side simplifies to \(x^2\), so the equation is \(x^2 = 7\), or \(x = \pm \sqrt{7}\). However, looking at the original equation, since the domain of \(\log_2\) is \((0, \infty)\), \(x\) can only be positive. Hence we only have one solution \(x = \sqrt{7}\).