Harvard-MIT Algebraic Geometry Seminar

How many points can a genus-2 curve have?

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Faltings proved that every curve C of genus $g > 1$ over a number field F has finitely many rational points. The upper bound on the number of points depends on C. Caporaso, Harris, and Mazur proved that Lang's Diophantine conjectures imply some remarkable bounds on these numbers: not only is there an upper bound $B(g, F)$ on $\#C(F)$, and thus also an upper bound $N(g, F)$ on the limsup of $\#C(F)$ as C ranges over genus-g curves over F, but the latter bound can be taken to be $N(g)$ independent of F. These bounds are all hopelessly ineffective even in the first case $(g, F) = (2, \mathbb{Q})$. We use a $K3$ surface of maximal Neron-Severi rank over $\mathbb{Q}$ to obtain some new records; for instance, $N(2, \mathbb{Q})$ is at least 150, and there are infinitely many genus-2 curves with a rational Weierstrass point and at least 118 other rational points over $\mathbb{Q}$.

Tuesday November 20th
3:00 p.m.
Harvard Science Center (507)