1. Let \( k \) be a field and \( R = k[x_1, \ldots, x_n] \). Verify the following statements from class.

   (a) Let \( \{I_\alpha\} \) be a collection of ideals in \( R \). Show that \( \bigcap_\alpha V(I_\alpha) = V(\bigcup_\alpha I_\alpha) \).

   (b) Let \( I, J \subset R \) be ideals. Show that \( V(IJ) = V(I) \cup V(J) \).

2. Let \( f \in k[x, y] \) be a polynomial of degree \( n > 0 \), and \( C = V(f) \). Let \( L \) be a line in \( \mathbb{A}_k^2 \) such that \( L \) is not contained in \( C \). Show that \( L \cap C \) is a finite set of no more than \( n \) points.

3. Show that the following are algebraic sets:

   (a) \( \{(t, t^2, t^3) \in \mathbb{A}_k^3 \mid t \in k\} \).

   (b) The set of \( m \times n \) matrices over \( k \) with rank \( \leq r \).

   (c) The set of points in \( \mathbb{A}_k^2 \) whose polar coordinates \((r, \theta)\) satisfy \( r = \sin(\theta) \).

   (d) \( V \times W \subset \mathbb{A}^{n+m} \), where \( V \) and \( W \) are algebraic sets in \( \mathbb{A}^n \) and \( \mathbb{A}^m \), respectively.

4. Show that the following are not algebraic sets:

   (a) \( \{(x, y) \in \mathbb{A}_k^2 \mid y = \sin(x)\} \)

   (b) The closed unit ball \( \{P \in \mathbb{A}_k^n \mid \|P\| \leq 1\} \).

5. Show that \( I = (x^2 + 1) \subset \mathbb{R}[x] \) is a radical ideal, but \( I \) is not the ideal of any set in \( \mathbb{A}^1_\mathbb{R} \).

6. Let \( J \subset k[x_1, \ldots, x_n] \) be an ideal and \( X \) and \( Y \) algebraic sets. Verify the following statements.

   (a) \( V(I(V(J))) = V(J) \).

   (b) \( I(V(I(X))) = I(X) \).

   (c) \( X = Y \) if and only if \( I(X) = I(Y) \).