1. (a) Define immersion.

(b) Define what it means for a property of smooth maps to be stable.

(c) Suppose $X$ is compact. Show that the property of being an immersion from $X$ to $Y$ is a stable property among maps from $X$ to $Y$.

2. Determine whether the equation(s) smoothly cut(s) out a manifold, and what the dimension of the manifold is if so:

(a) $xyz = 1$ in $\mathbb{R}^3$

(b) $x^3 - y^2 = 0$ in $\mathbb{R}^2$

(c) $x^2 - y^2 - z = 0$ and $4x - 2y - z = 3$ in $\mathbb{R}^3$

3. We say two maps $f : X \to Z$ and $f : Y \to Z$ are transversal if whenever $f(x) = g(y)$ we have $\text{im}(df_x) + \text{im}(dg_y) = TZ_{f(x)}$. Show that $f$ and $g$ are transversal if and only if $f \times g$ and $\Delta$ are transversal, where $f \times g : X \times Y \to Z \times Z$ is given by $(f \times g)(x, y) = (f(x), g(y))$ and $\Delta = \{(z, z) : z \in Z\} \subset Z \times Z$.

4. Let $X \subset \mathbb{R}^n$ be a compact submanifold of dimension $n - 1$ not containing the origin. Show that for almost every $v \in S^{n-1}$, the ray $R_v = \{tv : t \in \mathbb{R}, t \geq 0\}$ intersects $X$ in only finitely many points.

Remark (wouldn’t show up on a midterm): Can you treat the case when $X$ may contain the origin? Note this case is specific to submanifolds. If we had a map $f : X \to \mathbb{R}^n$, with $X$ of dimension $n - 1$, the statement in problem 4 goes through, but we wouldn’t be able to allow $f(X)$ to contain the origin.