1. [BN, 7.13] 
Show that \( f(z) = \int_{0}^{1} \frac{\sin(zt)}{t} \, dt \) is an entire function 
(a) by using Morera’s theorem 
(b) by obtaining a power series expansion

2. [BN, 7.14] 
Let \( f(z) \) be as given in problem 1. Show that \( f'(z) = \int_{0}^{1} \cos(zt) \, dt \) 
(a) using \( \frac{d}{dz} \left( \frac{\sin(zt)}{t} \right) = \cos(zt) \) 
(b) using power series

3. [BN, 9.9 and 10.1] For each of the following functions: find their singularities; classify them as removable, pole of order \( k \) [please give \( k \)], or essential; and determine the residue at each singularity.

(a) \( \frac{1}{z^4+z^2} \) 
(b) \( \cot(z) \) 
(c) \( \csc(z) \) 
(d) \( \frac{e^{1/z^2}}{z-1} \) 
(e) \( \frac{1}{z^2+3z+2} \) 
(f) \( \sin(1/z) \) 
(g) \( ze^{3/z} \) 
(h) \( \frac{1}{az^2+bz+c} \) for \( a \neq 0 \)

4. (a) [BN 9.10] Find the principal part of the Laurent expansion of \( f(z) = \frac{1}{(z^2+1)^2} \) about the point \( z = i \). 
(b) Compute \( \int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} \, dx \) 

Remark: Recall that the principal part of a Laurent expansion \( \sum_{-\infty}^{\infty} c_k z^k \) are the terms with negative exponent, i.e. \( \sum_{-\infty}^{-1} c_k z^k \).

5. [BN 9.11] Find the Laurent expansions for

(a) \( \frac{1}{z^4+z^2} \) about \( z = 0 \) 
(b) \( \frac{e^{1/z^2}}{z-1} \) about \( z = 0 \) 
(c) \( \frac{1}{z^2-1} \) about \( z = 2 \)