The Eigenvarieties workshop at the Clay Mathematics Institute: May 10-15

May 11, 2006

1 Where!

The Clay Mathematics Institute is at One Bow Street, Cambridge, MA

2 Schedule

- Wednesday May 10 9:30 a.m. - 5 p.m.
  - 9:30 a.m.- 10:30 a.m. Coffee
  - 10:30-12:00 Joel Bellaiche
  - 1:30-3:00 p.m. Eric Urban
  - 4 p.m. - 5 p.m. Jeremy Teitelbaum
- Thursday May 11 9:30 a.m. - 5 p.m.
  - 9:30 a.m.- 10:30 a.m. Coffee
  - 10:30-12:00 Mark Kisin
  - 1:30-3:00 p.m. Chris Skinner
- Friday May 12 9:30 a.m. - 5 p.m.
  - 9:30 a.m.- 10:30 a.m. Coffee
  - 10:30-12:00 Joel Bellaiche
  - 1:30-3:00 p.m. Eric Urban
  - 4:00-5:00p.m. Informal Question Session headed by Robert Pollack and Fred Diamond
- Saturday May 13 1:30 p.m. - 5 p.m.
  - 1:30-3:00 p.m. Chris Skinner

- 3 p.m.-4 p.m. Mark Kisin

- Sunday May 14 1:30 p.m. - 5 p.m.
  - 1:30-3:00 p.m. Eric Urban
  - 3 p.m.-4 p.m. Mark Kisin

- Monday May 15 9:30 a.m. - 5 p.m.
  - 9:30 a.m.- 10:30 a.m. Coffee
  - 10:30-12:00 Joel Bellaiche
  - 1:30 p.m. - 3:00 p.m. Mark Kisin

3 Background material for some of the lectures

- **The series of lectures by Joel Bellaiche**

  Joel Bellaiche writes:

  - **First talk: "Residually multiplicity-free pseudocharacters."**

    This would be a purely algebraic talk without any reference to automorphic forms nor Galois theory (or only as examples). The aim would be to fully explain our understanding of pseudocharacters $T : R \to A$ where A is a (strict) local henselian ring, under the hypothesis that T is residually without multiplicity. After recalling our basic structure theorem on the algebra $R/\ker T$ and its consequence on reducibility loci and extensions (that Gaetan did explain) I will state and prove two results : that $R/\ker T$ can be embeeded in a matrix algebra over a suitable ring (generalizing older results of Procesi) and a necessary and sufficient condition on A for every pseudocharacter as above to come from a true representation over A (the criterion is : A is a UFD). I will give some applications (for example on limit of sequences of representations). I will also explain why those results fail for non residually multiplicity free pseudocharacters and what weaker results may be expected (and in some cases proved) in the general case.

  - **Second talk : "Generalization of a result of Kisin on existence of crystalline periods in a family."**

    The main result of Kisin's Inventiones paper on the Fontaine-Mazur conjecture was a theorem about existence of crystalline periods in a rigid analytic family of Galois representations. Roughly speaking, if in such a family, there is a Zariski dense set of representations that are crystalline and that have a crystalline Frobenius eigenvalue that can be interpolated to a rigid analytic function on the whole family, then there is a crystalline period on the whole family (and in particular, on every point and on every thickened point) that is an eigenvector for the crystalline Frobenius with this function as eigenvalue. In Kisin's work "family" mean free module with a Galois action. The aim of this talk is to explain the generalization of this result for families defined by general Galois pseudo-characters.

– **Third talk : "Arthur conjectures and A-packets for unitary groups."**

This talk would be mainly expository, with very few new results. I plan to explain the conjectures of Arthur, and the formalism of L and A-parameter, and L and A-packets and in particular how they predict the existence of non-tempered representation well-suited for trying to prove the Bloch-Kato conjecture (representations that are used in my thesis, the work of Urban-Skinner and my work with Gaetan). I will focus on the case of unitary groups and explain in detail the case of $U(3)$ that is completely (non-conjecturally) understood thanks to Rogawski. I will then explain my result on compatibility between local and global Langlands correspondance for this group."

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• **The series of lectures by Eric Urban (p-adic trace formulas and Eigenvarieties for unitary groups).**

Eric Urban writes:

"The purpose of this topic is to present the construction of the Eigenvariety for any unitary groups.

Instead of constructing a big space interpolating the spaces of p-adic modular forms of a given weight when the weight varies I show that one can interpolate the traces of compact Hecke operators acting on these spaces. This approach is analogous to constructing families of Galois representation by interpolating the trace (i.e. what is called pseudo-representation by Wiles). As a by product of this approach, one can prove

1. a p-adic trace formula that gives a tool to show transfer of p-adic modular forms between different groups.
2. p-adic deformations of Eisenstein series and Selmer groups. (This is a joint ongoing work with Chris Skinner):

   Let ρ be a p-adic Galois representation of G_K associated to a cuspidal representation of a unitary group (defined from an hermitian form over an imaginary quadratic field K) such that

   (a) $\rho^\vee(1) = \rho^c$,

   (b) $L(\rho, s)$ vanishes at $s = 0$,

   We show (assuming some standard conjectures on the existence of Galois representations for unitary groups) that $H^1_f(K, \rho)$ is of rank at least one. Here the usual sign condition in our original work with Chris and also in Bellaiche-Chenevier's paper that $\epsilon = -1$ is no longer necessary.

   I believe but one (probably !) needs to check all the steps of the proof that in the case $\epsilon = 1$ and $L(\rho, 0) = 0$ we can show that the rank of the Selmer group is at least two. If it does work, it should give a strategy for higher order of vanishing for the p-adic L-functions..."

• **The lecture series by Mark Kisin "The eigencurve via Galois representations"**

1. Coleman-Mazur: The Eigencurve, in *Galois representations in Arithmetic Algebraic Geometry (Durham 1996)* London Math Soc Lecture Notes **154** (1997) 1-113

2. Mark Kisin: "Overconvergent modular forms and the Fontaine-Mazur conjecture. Invent. Math. **153**(2) (2003), 373-454
3. Modularity and patching local rings over global ones.

   Reading: Sections 3.2-3.5 of the preprint “Moduli of finite flat group schemes and modularity” on Kisin’s homepage.
4. Existence of semi-stable deformation rings and the Breuil-Mezard conjecture.

   Reading: For the first part of this topic there should soon be a preprint. The construction uses the theory of the paper "Crystalline representations and F-crystals” which is on Kisin’s webpage.
5. The Fontaine-Mazur conjecture via the p-adic local Langlands correspondence.

The series of lectures by Chris Skinner.

Chris Skinner writes: “Probably the most important thing is to understand the structure of Ribet’s paper and how Wiles incorporated it with Hida familes to prove the Main Conjecture. The rest is just for those interested in details of the techniques.” Specific references for the specific topics to be covered are:

- 1. *General Iwasawa theory:*
   (a) Greenberg’s papers, esp.
   “Iwasawa theory for p-adic representations.” Algebraic number theory, 97–137, Adv. Stud. Pure Math., 17, Academic Press, Boston, MA, 1989. (Reviewer: Karl Rubin) 11R23 (11G40 11R34)
   (b) “Iwasawa theory for motives. L-functions and arithmetic.” (Durham, 1989), 211–233, London Math. Soc. Lecture Note Ser., **153**, Cambridge Univ. Press, Cambridge, 1991.
   (c) “Iwasawa theory and p-adic deformations of motives.” Motives (Seattle, WA, 1991), 193–223, Proc. Sympos. Pure Math., **55**, Part 2, Amer. Math. Soc., Providence, RI, 1994.

2. *for the* GL(1)/GL(2) *cases:*
   (a) Ribet, Kenneth A. “A modular construction of unramified p-extensions of $\mathbf{Q}(\mu_p)$.” Invent. Math. **34** (1976), no. 3, 151–162.
   (b) Wiles, A. “The Iwasawa conjecture for totally real fields.” Ann. of Math. (2) **131** (1990), no. 3, 493–540.
   (c) Wiles, A. “On p-adic representations for totally real fields.” Ann. of Math. (2) **123** (1986), no. 3, 407–456.

3. *for Hida theory:*

   Hida’s papers, esp.

   “p-adic automorphic forms on reductive groups.” Automorphic forms. I. Astrisque No. **298** (2005), 147–254.(also available on his webpage)

4. *for Galois representations for unitary groups:*
   – See the example of unitary groups in
   Blasius, Don; Rogawski, Jonathan D. Zeta functions of Shimura varieties. Motives (Seattle, WA, 1991), 525–571, Proc. Sympos. Pure Math., **55**, Part 2, Amer. Math. Soc., Providence, RI, 1994.

- Also see the relevant section(s) of
  Harris, Michael: "L-functions and periods of polarized regular motives." J. Reine Angew. Math. **483** (1997), 75–161.

5. *or the Eisenstein series on unitary groups see (at your own risk):*

   Shimura, Goro Euler products and Eisenstein series. CBMS Regional Conference Series in Mathematics, **93**.

   esp. chapter 12 and chapters 20-22.