COMPLEX EIGENVALUES

Math 21b, O. Knill

Gauss published the first correct proof of the fundamental theorem of algebra in his doctoral thesis, but still claimed in 1825 that the true metaphysics of the square root of $-1$ is elusive as late as 1825. By 1831 Gauss overcame his uncertainty about complex numbers and published his work on the geometric representation of complex numbers as points in the plane. In 1797, a Norwegian Caspar Wessel (1745-1818) and in 1806 a Swiss clerk named Jean Robert Argand (1768-1822) (who stated the theorem the first time for polynomials with complex coefficients) did similar work. But these efforts went unnoticed. William Rowan Hamilton (1805-1865) (who would also discover the quaternions while walking over a bridge) expressed in 1833 complex numbers as vectors.

Complex numbers continued to develop to complex function theory or chaos theory, a branch of dynamical systems theory. Complex numbers are helpful in geometry in number theory or in quantum mechanics. Once believed fictitious they are now most "natural numbers" and the "natural numbers" themselves are in fact the most "complex." A philosopher who asks "does $\sqrt{-1}$ really exist?" might be shown the representation of $x+iy$ as $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$. When adding or multiplying such dilation-rotation matrices, they behave like complex numbers: for example $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ plays the role of $i$.

Gauss published the first correct proof of the fundamental theorem of algebra in his doctoral thesis, but still claimed in 1825 that the true metaphysics of the square root of $-1$ is elusive as late as 1825. By 1831 Gauss overcame his uncertainty about complex numbers and published his work on the geometric representation of complex numbers as points in the plane. In 1797, a Norwegian Caspar Wessel (1745-1818) and in 1806 a Swiss clerk named Jean Robert Argand (1768-1822) (who stated the theorem the first time for polynomials with complex coefficients) did similar work. But these efforts went unnoticed. William Rowan Hamilton (1805-1865) (who would also discover the quaternions while walking over a bridge) expressed in 1833 complex numbers as vectors.

Complex numbers continued to develop to complex function theory or chaos theory, a branch of dynamical systems theory. Complex numbers are helpful in geometry in number theory or in quantum mechanics. Once believed fictitious they are now most "natural numbers" and the "natural numbers" themselves are in fact the most "complex." A philosopher who asks "does $\sqrt{-1}$ really exist?" might be shown the representation of $x+iy$ as $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$. When adding or multiplying such dilation-rotation matrices, they behave like complex numbers: for example $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ plays the role of $i$.

For decades, many mathematicians still thought complex numbers were a waste of time. Others used complex numbers extensively in their work. In 1620, Girard suggested that an equation may have as many roots as its degree in 1620. Leibniz (1646-1716) spent quite a bit of time trying to apply the laws of algebra to complex numbers. He and Johann Bernoulli used imaginary numbers as integration aids. Lambert used complex numbers for map projections, d’Alembert used them in hydrodynamics, while Euler, D’Alembert and Lagrange used them in their incorrect proofs of the fundamental theorem of algebra. Euler wrote first the symbol $i$ for $\sqrt{-1}$.

Gauss published the first correct proof of the fundamental theorem of algebra in his doctoral thesis, but still claimed in 1825 that the true metaphysics of the square root of $-1$ is elusive as late as 1825. By 1831 Gauss overcame his uncertainty about complex numbers and published his work on the geometric representation of complex numbers as points in the plane. In 1797, a Norwegian Caspar Wessel (1745-1818) and in 1806 a Swiss clerk named Jean Robert Argand (1768-1822) (who stated the theorem the first time for polynomials with complex coefficients) did similar work. But these efforts went unnoticed. William Rowan Hamilton (1805-1865) (who would also discover the quaternions while walking over a bridge) expressed in 1833 complex numbers as vectors.

Complex numbers continued to develop to complex function theory or chaos theory, a branch of dynamical systems theory. Complex numbers are helpful in geometry in number theory or in quantum mechanics. Once believed fictitious they are now most "natural numbers" and the "natural numbers" themselves are in fact the most "complex." A philosopher who asks "does $\sqrt{-1}$ really exist?" might be shown the representation of $x+iy$ as $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$. When adding or multiplying such dilation-rotation matrices, they behave like complex numbers: for example $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ plays the role of $i$.

Gauss published the first correct proof of the fundamental theorem of algebra in his doctoral thesis, but still claimed in 1825 that the true metaphysics of the square root of $-1$ is elusive as late as 1825. By 1831 Gauss overcame his uncertainty about complex numbers and published his work on the geometric representation of complex numbers as points in the plane. In 1797, a Norwegian Caspar Wessel (1745-1818) and in 1806 a Swiss clerk named Jean Robert Argand (1768-1822) (who stated the theorem the first time for polynomials with complex coefficients) did similar work. But these efforts went unnoticed. William Rowan Hamilton (1805-1865) (who would also discover the quaternions while walking over a bridge) expressed in 1833 complex numbers as vectors.

Complex numbers continued to develop to complex function theory or chaos theory, a branch of dynamical systems theory. Complex numbers are helpful in geometry in number theory or in quantum mechanics. Once believed fictitious they are now most "natural numbers" and the "natural numbers" themselves are in fact the most "complex." A philosopher who asks "does $\sqrt{-1}$ really exist?" might be shown the representation of $x+iy$ as $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$. When adding or multiplying such dilation-rotation matrices, they behave like complex numbers: for example $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ plays the role of $i$.

For decades, many mathematicians still thought complex numbers were a waste of time. Others used complex numbers extensively in their work. In 1620, Girard suggested that an equation may have as many roots as its degree in 1620. Leibniz (1646-1716) spent quite a bit of time trying to apply the laws of algebra to complex numbers. He and Johann Bernoulli used imaginary numbers as integration aids. Lambert used complex numbers for map projections, d’Alembert used them in hydrodynamics, while Euler, D’Alembert and Lagrange used them in their incorrect proofs of the fundamental theorem of algebra. Euler wrote first the symbol $i$ for $\sqrt{-1}$.

For decades, many mathematicians still thought complex numbers were a waste of time. Others used complex numbers extensively in their work. In 1620, Girard suggested that an equation may have as many roots as its degree in 1620. Leibniz (1646-1716) spent quite a bit of time trying to apply the laws of algebra to complex numbers. He and Johann Bernoulli used imaginary numbers as integration aids. Lambert used complex numbers for map projections, d’Alembert used them in hydrodynamics, while Euler, D’Alembert and Lagrange used them in their incorrect proofs of the fundamental theorem of algebra. Euler wrote first the symbol $i$ for $\sqrt{-1}$.