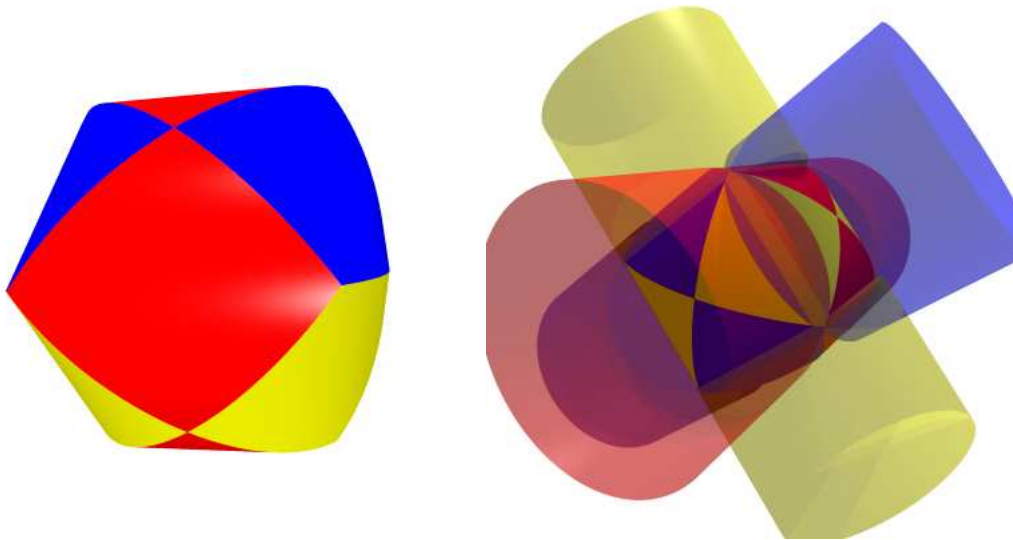


## THREE CYLINDERS

Maths21a

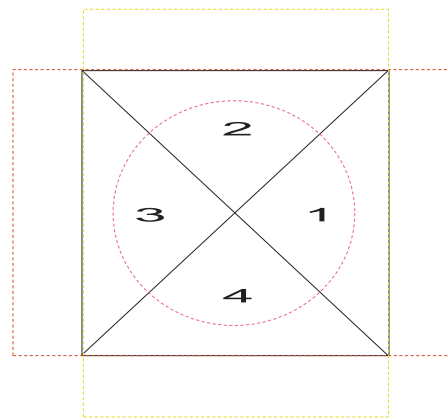
Problem: Find the body obtained by intersecting the three solid cylinders  $x^2 + z^2 \leq 1$ ,  $y^2 + z^2 \leq 1$  and  $x^2 + y^2 \leq 1$ .



The problem gets easier with the advise:

- make a good picture of the situation
- use symmetry to simplify the problem.

We first look at the intersection  $R$  of the cylinders around the  $x$ -axes and the  $y$ -axis from above. You see 4 equal parts. When considering the part above and below the  $xy$ -plane separately, one can find an integral which is  $1/8$  of the answer.



The third cylinder  $x^2 + y^2 \leq 1$  appears as a circle in this picture.

To compute the volume, we work only in one quarter of the body and where  $z > 0$ . The radius goes from 0 to  $a$ , the angle from 0 to  $\pi/4$ . The equation of the roof of the body is  $z = \sqrt{1 - y^2}$ , because in region 1, only the cylinder  $y^2 + z^2 = 1$  matters. In cylindrical coordinates, not forgetting the integration factor  $r$ , and  $y = r \sin(\theta)$  and to multiply everything by 8.

We use cylindrical coordinates

$$8 \int_{-\pi/4}^{\pi/4} \int_0^1 \sqrt{1 - r^2 \sin^2(t)} r \, dr \, dt = -16/3 + 8\sqrt{2}$$