COORDINATES/DISTANCES

O. Knill, Maths21a

**PARITY.** We usually work with a right handed coordinate system. The photographers coordinate system is an example of a left handed coordinate system. The "right hand rule": thumb=x-direction index finger=y-direction and middle finger=z-direction allows to check that the coordinate system is "right handed".

Parity is relevant in biology (orientation of DNA or Proteins) or particle physics, ("parity violation": physical laws change when we look at them in the mirror). Coordinate systems with different parity can not be rotated into each other.

**GEOMETRICAL OBJECTS.** curves, surfaces and bodies are examples of geometrical objects which can be described using functions of several variables. We look at some of them here to get some feel about space. The objects will be treated in more detail later.

**SPHERE.** A sphere of radius \( r \) centered at \( P = (a, b, c) \) is the collection of points which have the distance \( r \) from \( P \). The equation of a sphere is

\[
(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2.
\]

**COMPLETION OF SQUARE.** The equation \( x^2 + bx + c = 0 \) is solved by adding \((b/2)^2\) on both sides. This is the "completion of the square".

\[
(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2 - (b/2)^2 - c = r^2 - b/2.
\]

**CENTER AND RADIUS OF A SPHERE.** The equation \( x^2 + 5x + y^2 - 2y + z^2 = -1 \) is after completion of the square in each variable equivalent to \((x + 5/2)^2 - 25/4 + (y - 1)^2 + (y - 1)^2 + z^2 = (5/2)^2\). The equation describes therefore a sphere with center \((5/2, 1, 0)\) and radius \(5/2\).

**COORDINATE PLANES, QUADRANTS, OCTANTS.** The coordinate axis \( x = 0, y = 0 \) or \( z = 0 \) divide the space into 8 regions called octants. This could be continued into higher dimensions: how many "hyper-regions" are there in four dimensional "hyper-space" which is labeled by points with 4 coordinates \((x, y, z, w)\)? There are 16 hyper-regions and each of them contains one of the 16 points \((x, y, z, w)\), where \(x, y, z, w\) are either +1 or −1.

**DESCRIBING PLANES.** To draw the set of all points \((x, y, z)\) which satisfy \( x + 2y - 3z = 2 \), we first find the intersections with the three coordinate axes. These intersects \( P = (2, 0, 0), Q = (0, 1, 0), R = (0, 0, -2/3) \). Then we draw the traces, the intersections of the set with the coordinate planes \(x = 0, y = 0\) or \( z = 0\). These three lines bound a triangle in space. Drawing this triangle indicates well the position of the plane.

**HISTORICAL.** In an appendix to "Geometry" to his "Discours de la méthode" René Descartes (1596-1650) promoted the idea that algebra could be used as a general method to solve geometric problems. In honor of Descartes, the rectangular coordinate system is today called the Cartesian coordinate system.

**Anecdote:** In 1649, Queen Christina of Sweden persuaded Descartes to go to Stockholm. However the Queen wanted to draw tangents at 5 a.m. and Descartes broke the habit of his lifetime of getting up at 11 o’clock. After only a few months in the cold northern climate, walking to the palace for 5 o’clock every morning, Descartes died of pneumonia.