The Mathematician Charles W. Cobb and the economist Paul H. Douglas (picture) found in 1928 empirically a formula \( F(L, K) = bL^\alpha K^\beta \) which gives the total production \( F \) of an economic system as a function of the amount of labor \( L \) and the capital investment \( K \).

By fitting data, they got \( b = 1.01, \alpha = 0.75, \beta = 0.25 \). By rescaling the production unit we can get \( b = 1 \) and work with the formula:

\[ F(L, K) = L^{3/4}K^{1/4} \]

Assume that the labor and capital investment are bound by the constraint \( G(L, K) = L^{3/4} + K^{1/4} = 1 \). Where is the production \( P \) maximal under this constraint?

\[
\nabla F(L, K) =
\]

\[
\nabla G(L, K) =
\]

Solve: \( \nabla F(L, K) = \lambda \nabla G(L, K), G(L, K) = 1 \):