Asteroids approximately move on elliptical paths \( r(t) = (a \cos(t), b \sin(t)) \) around the sun. If we calculate the length of the ellipse for general parameters \( a, b \), we obtained the formula

\[
\int_0^{2\pi} \sqrt{a^2 \sin^2(t) + b^2 \cos^2(t)} \, dt .
\]

Unfortunately, we can not solve this so called ”elliptic” integral.

To get an approximation, we write

\[
\int_0^{2\pi} a \sqrt{\sin^2(t) + \cos^2(t) - (1 - b^2/a^2) \cos^2(t)} \, dt = a \int_0^{2\pi} \sqrt{1 - \epsilon^2 \cos^2(t)} \, dt ,
\]

where \( \epsilon = \sqrt{1 - b^2/a^2} \) is called the eccentricity of the ellipse. We have a linear approximation

\[
\sqrt{1 - \epsilon^2 \cos^2(t)} \sim 1 - (\epsilon^2/2) \cos^2(t)
\]

from the Taylor series.

An approximation for the length of the ellipse is therefore

\[
\int_0^{2\pi} a (1 - \epsilon^2/2) \sin^2(t) \, dt .
\]

Calculate an approximation of the ellipse length with this formula.

BBC News Online:

Thursday, 20 June, 2002, 16:29 GMT 17:29 UK

Space rock’s close approach

By Dr David Whitehouse
BBC News Online science editor

Astronomers have revealed that on 14 June, an asteroid the size of a football pitch made one of the closest ever recorded approaches to the Earth.

It is only the sixth time an asteroid has been seen to penetrate the Moon’s orbit, and this is by far the biggest rock to do so.

What has worried some astronomers, though, is that the space object was only detected on 17 June, several days after its flyby.

It was found by astronomers working on the Lincoln Laboratory Near Earth Asteroid Research (Linear) search programme in New Mexico.
Catalogued as 2002MN, the asteroid was traveling at over 10 kilometers a second (23,000 miles per hour) when it passed Earth at a distance of around 120,000 km (75,000 miles).

The last time such an object is recorded to have come this close was in December 1994.

'Wake up call'

The space rock has a diameter of between 50-120 metres (160 - 320 feet). This is actually quite small when compared with many other asteroids and incapable of causing damage on a global scale.

Nonetheless, an impact from such a body would still be dangerous.

If 2002MN had hit the Earth, it would have caused local devastation similar to that which occurred in Tunguska, Siberia, in 1908, when 2,000 square kilometers of forest were flattened.

Dr Benny Peiser, of Liverpool John Moores University, UK, told BBC News Online: "Our ever increasing observational capacity is now detecting these close shaves from small objects.

"The probability is actually quite high that a Tunguska-sized object will hit us in our lifetimes."

'Bolt from the blue'

A major issue of concern centers on how late this object was picked up.

Dr John Davies, of the Royal Observatory Edinburgh, has calculated the orbit of the asteroid from the Linear data.

He concludes that the asteroid came out of the Sun and was impossible for Linear to see until one hour after its flyby of the Earth on the 14th.

Dr Davies said: "...if an asteroid were to approach close to an imaginary line joining the Earth and the Sun it would never be visible in a night-time sky and would be quite impossible to discover with normal telescopes. Its arrival would come, literally, as a bolt from the blue."

Space-based telescopes, such as Hubble and the future European Gaia spacecraft, are the only means of searching for asteroids in the daytime sky.