Name:

- Please mark the box to the left which lists your section and make sure you have written down your name in the box above.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which the grader cannot read will not receive credit.
- Except for the TF and matching problem, show your work.
- No notes, books, calculators, computers, or other electronic aids can be used.
- You have 90 minutes time to complete your work.

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<td>MWF 10 Samik Basu</td>
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<td>MWF 10 Joachim Krieger</td>
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<td>MWF 11 Matt Leingang</td>
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<td>MWF 11 Veronique Godin</td>
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<td>TTH 10 Oliver Knill</td>
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<td>TTH 115 Thomas Lam</td>
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Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T  F  
The vectors \( \langle 1, 2, 1 \rangle \) and \( \langle 3, 2, -5 \rangle \) are perpendicular.

2) T  F  
\[ |\vec{v} \times \vec{w}| = |v||w|\cos(\alpha), \] where \( \alpha \) is the angle between \( \vec{v} \) and \( \vec{w} \).

3) T  F  
The vector \( \vec{i} \times (\vec{j} \times \vec{k}) \) has length 1.

4) T  F  
The distance between the \( z \)-axis and the line \( x - 1 = y = 0 \) is 1.

5) T  F  
If two vectors \( \vec{v} \) and \( \vec{w} \) are perpendicular, then the lengths of \( \vec{v} + \vec{w} \) and \( \vec{v} - \vec{w} \) are the same.

6) T  F  
If \( \vec{r}_1(t) \) is a parameterization of a curve and \( \vec{r}_2(t) \) is a second parameterization of the same curve and \( \vec{r}_1(0) = \vec{r}_2(0) \), then the velocity vectors \( \vec{r}_1'(t) \) and \( \vec{r}_2'(t) \) are the same.

7) T  F  
There is a surface which has both hyperbola and parabola as traces.

8) T  F  
The parameterization \( \vec{r}(\theta, \phi) = \langle 5 \cos(\theta) \sin(\phi), 2 \sin(\theta) \sin(\phi), 3 \cos(\phi) \rangle \) with \( \theta \in [0, 2\pi) \), \( \phi \in [0, 2\pi] \) describes an ellipsoid.

9) T  F  
If the velocity vector \( \vec{r}'(t) \) is perpendicular to the vector \( \vec{r}(t) \), then the parametrized curve \( \vec{r}(t) \) is on a sphere.

10) T  F  
The function \( f(x, y) = 2x^2y^2/(x^2+y^2) \) with \( f(0, 0) = 0 \) is continuous in the entire plane.

11) T  F  
The function \( f(x, y) = x^{\sin(x)} + \cos(xy^6) \) satisfies the partial differential equation \( f_{xxyy}(x, y) = f_{xyyx}(x, y) \) everywhere in the plane.

12) T  F  
The surface \( f(x, y, z) = x^2 + y^2 - z^2 = -1 \) is a one-sheeted hyperboloid.

13) T  F  
The curvature of the circle \( x^2 + y^2 = 4 \) is 2.

14) T  F  
The equation \( x^2 + y^2/4 = 1 \) in space describes an ellipsoid.

15) T  F  
For any two vectors \( \vec{v}, \vec{w} \) we have \( \text{proj}_{\vec{v}}(\vec{w}) = \text{proj}_{\vec{w}}(\vec{v}) \).

16) T  F  
The set of points in space which have distance 1 from a line form a cylinder.

17) T  F  
The surface given in spherical coordinates as \( \theta = \pi/3 \) is a half cone.

18) T  F  
The velocity vector of a parametric curve \( \vec{r}(t) \) always has constant length.

19) T  F  
If \( f \) satisfies the PDE \( f_{xx} = f_{tt} \), then \( g(t, x) = f(t + x, t - x) \) satisfies the PDE \( g_{tx} = 0 \).

20) T  F  
The volume of a parallelepiped spanned by \( \vec{u}, \vec{v}, \vec{w} \) is \( |(\vec{u} \times \vec{v}) \times \vec{w}| \).
Problem 2) (10 points)

Match the equation with the pictures. No justifications are necessary in this problem.

<table>
<thead>
<tr>
<th>Enter I,II,III,IV,V,VI here</th>
<th>Equation</th>
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<tbody>
<tr>
<td>I</td>
<td>$x^4 + y^4 + z^4 - 1 = 0$</td>
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<td>$-x^2 + y^2 - z^2 - 1 = 0$</td>
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<td>$x^2 + z^2 = 1$</td>
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<tr>
<td>II</td>
<td>$-y^2 + z^2 = 0$</td>
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<tr>
<td></td>
<td>$x^2 - y^2 + 3z^2 - 1 = 0$</td>
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<tr>
<td>III</td>
<td>$x^2 - y - z^2 = 0$</td>
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Problem 3) (10 points)

a) (5 points) Find the distance of the point $P = (1, 2, 3)$ to the plane $x + y + z = 1$.

b) (5 points) Find the distance of the point $P = (1, 2, 3)$ to the line $x - 1 = y - 3 = z - 4$.

Problem 4) (10 points)

a) (4 points) Write down the parameterization of the sphere $(x - 1)^2 + (y - 1)^2 + (z + 2)^2 = 9$. 
using suitably centered spherical coordinates $\theta$ and $\phi$.

b) (3 points) The latitudes on the sphere are curves defined by the equation $\phi = \text{constant}$. Write down the parametric equations for the latitude $\phi = \pi/4$.

c) (3 points) Write down the arc length integral for this curve and evaluate it.

**Problem 5) (10 points)**

Given the plane $x + y + z = 6$ containing the point $P = (2, 2, 2)$. Given is also a second point $Q = (3, -2, 2)$.

a) (5 points) Find the equation $ax + by + cz = d$ for the plane through $P$ and $Q$ which is perpendicular to the plane $x + y + z = 6$.

b) (5 points) Find the symmetric equation for the intersection of these two planes.

![Image of planes intersecting]

**Problem 6) (10 points)**

Intersecting the elliptic cylinder $x^2 + y^2/4 = 1$ with the plane $z = \sqrt{3}x$ gives a curve in space.

a) (3 points) Verify that this curve is parametrized by $\vec{r}(t) = \langle \sin(t), 2\cos(t), \sqrt{3}\sin(t) \rangle$ and give the parameter interval.

b) (3 points) Compute the unit tangent vector $\vec{T}$ to the curve at the point $(0, 2, 0)$. 

![Image of elliptic cylinder and plane intersecting]
c) (4 points) Write down the arc length integral and evaluate the arc length of the curve.

**Problem 7) (10 points)**

We know the acceleration \( \vec{r}''(t) = \langle 2, 1, 3 \rangle + t \langle 1, -1, 1 \rangle \) and the initial position \( \vec{r}(0) = \langle 0, 0, 0 \rangle \) and initial velocity \( \vec{r}'(0) = \langle 0, 0, 0 \rangle \) of an unknown curve \( \vec{r}(t) \). Find \( \vec{r}(100) \).

**Problem 8) (10 points)**

The elliptic paraboloid \( f(x, y, z) = x^2 + 2y^2 - z = 0 \) contains the point \( (1, 1, 3) \).

a) (4 points) Find the equation for the tangent plane at \( (1, 1, 3) \).

b) (3 points) Write down the linear approximation function \( L(x, y, z) \) of \( f(x, y, z) \) at \( (1, 1, 3) \).

c) (3 points) Estimate \( f(1.01, 1.0002, 2.999) \).

**Problem 9) (10 points)**

a) (5 points) Show that the function \( u(t, x) = \cos(2t - x) + \sin(2t - x) \) satisfies the **Klein-Gordon partial differential equation**

\[
    u_{tt} = u_{xx} - 3u .
\]

b) (3 points) Describe the level curves of \( u \).

c) (2 points) Parameterize one of the level curves \( u(t, x) = 1 \).