1/17/2007, FIRST PRACTICE FINAL **Math 21a, Fall 2007**

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- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Show your work. Answers without reasoning can not be given credit except for the True/False and multiple choice problems.
- Please write neatly.
- Do not use notes, books, calculators, computers, or other electronic aids.
- Unspecified functions are assumed to be smooth and defined everywhere unless stated otherwise.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
14		10
Total:		150

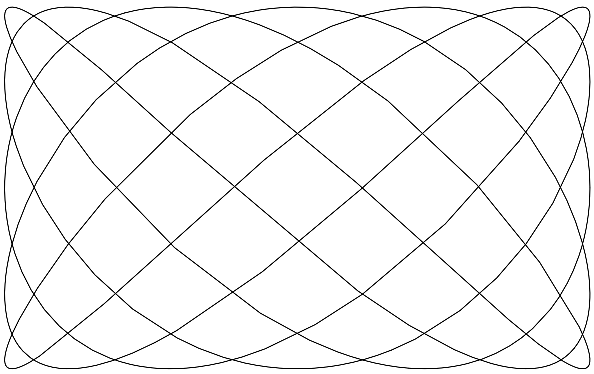
Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

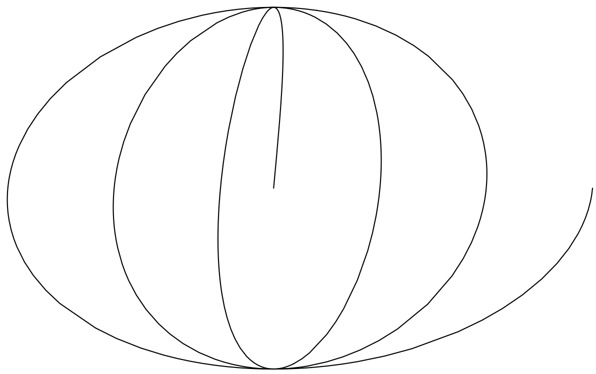
1)	T	F	The length of the curve $\vec{r}(t) = \langle \sin(t), t^4 + t, \cos(t) \rangle$ on $t \in [0,1]$ is the same as the length of the curve $\vec{r}(t) = \langle \sin(t^2), t^8 + t^2, \cos(t^2) \rangle$ on $[0,1]$.
2)	T	F	The parametric surface $\vec{r}(u,v) = (5u - 3v, u - v - 1, 5u - v - 7)$ is a plane.
3)	T	F	Any function $u(x,y)$ that obeys the differential equation $u_{xx} + u_x - u_y = 1$ has no local maxima.
4)	T	F	The length of the vector $\vec{b}$ onto a vector $\vec{a}$ is smaller or equal than the length of the vector $\vec{b}$.
5)	T	F	If $f(x,y)$ is a function such that $f_x - f_y = 0$ then f is called conservative.
6)	T	F	$(\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{u} \times \vec{w}) \cdot \vec{v}$ for all vectors $\vec{u}, \vec{v}, \vec{w}$.
7)	T	F	The equation $\rho = \phi/4$ in spherical coordinates is half a cone.
8)	T	F	If $f(x,y) = \frac{x^3}{x^2+y^2}$ and $(x,y) \to (0,0)$ then $f(x,y) \to 0$.
9)	T	F	$\int_0^1 \int_0^x 1 \, dydx = 1/2$.
10)	T	F	Let $\vec{a}$ and $\vec{b}$ be two vectors which are perpendicular to a given plane Σ. Then $\vec{a} + \vec{b}$ is also perpendicular to Σ.
11)	T	F	If $g(x,t) = f(x - vt)$ for some function f of one variable $f(z)$ then g satisfies the differential equation $g_{tt} - v^2 g_{xx} = 0$.
12)	T	F	If $f(x,y)$ is a continuous function on $\mathbf{R}^2$ such that $\int\int_D f \, dA \geq 0$ for any region D then $f(x,y) \geq 0$ for all (x,y).
13)	T	F	Assume the two functions $f(x,y)$ and $g(x,y)$ have both the critical point $(0,0)$ which are saddle points, then $f + g$ has a saddle point at $(0,0)$.
14)	T	F	If $f(x,y)$ is a function of two variables and if $h(x,y) = f(g(y), g(x))$, then $h_x(x,y) = f_y(g(y), g(x))g'(y)$.
15)	T	F	If we rotate a line around the z axis, we obtain a cylinder.
16)	T	F	If $u(x,y)$ satisfies the transport equation $u_x = u_y$ everywhere in the plane, then the vector field $\vec{F}(x,y) = \langle u(x,y), u(x,y) \rangle$ is a gradient field.
17)	T	F	$3 \, \text{grad}(f) = \frac{d}{dt} f(x+t, y+t, z+t)$.
18)	T	F	If $\vec{F}$ is a vector field in space and f is equal to the line integral of $\vec{F}$ along the straight line C from $(0,0,0)$ to (x,y,z), then $\nabla f = \vec{F}$.
19)	T	F	The line integral of $\vec{F}(x,y) = (x,y)$ along an ellipse $x^2 + 2y^2 = 1$ is zero.
20)	T	F	The identity $\text{div}(\text{grad}(f)) = 0$ is always true.

Problem 2) (10 points)

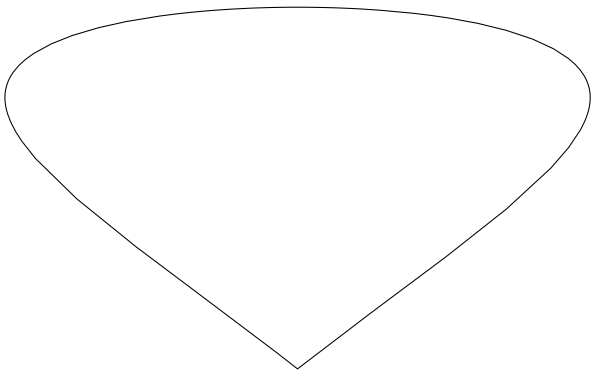
Match the equations with the curves. No justifications are needed.



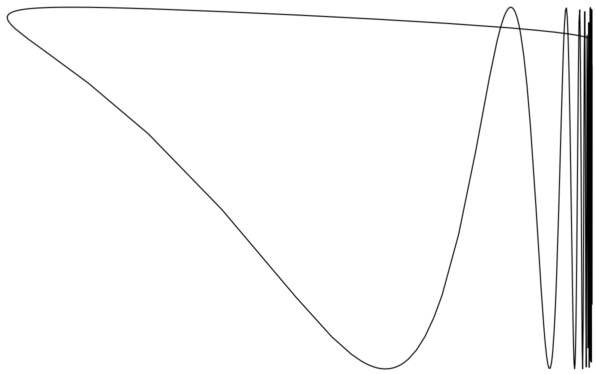
I



II



III



IV

Enter I,II,III,IV here	Equation
	$\vec{r}(t) = (\sin(t), t(2\pi - t))$
	$\vec{r}(t) = (\cos(5t), \sin(7t))$
	$\vec{r}(t) = (t\cos(t), \sin(t))$
	$\vec{r}(t) = (\cos(t), \sin(6/t))$

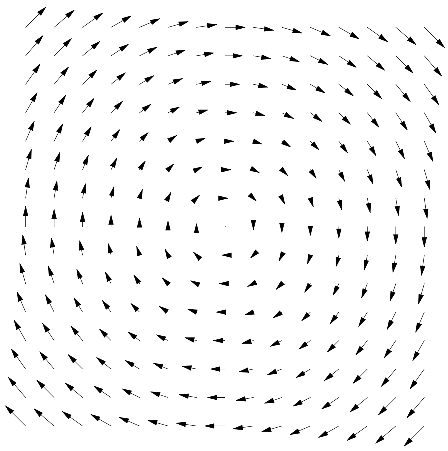
Problem 3) (10 points)

In this problem, vector fields F are written as $F = \langle P, Q \rangle$. We use abbreviations $\text{curl}(F) = N_x - M_y$ and $\text{div}(F) = M_x + N_y$. When stating $\text{curl}(F)(x,y) = 0$ we mean that $\text{curl}(F)(x,y) = 0$ vanishes for **all** (x,y). The statement $\text{curl}(F) \neq 0$ means that $\text{curl}(F)(x,y)$ does not vanish for at least one point (x,y).

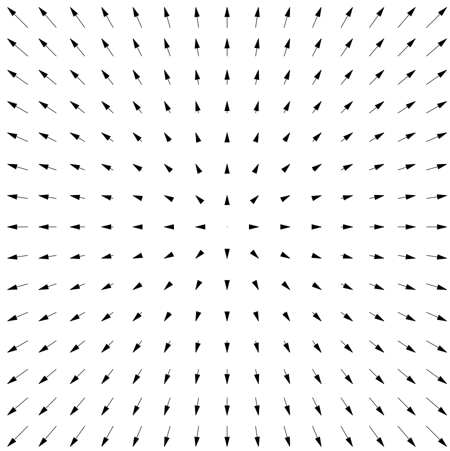
The same remark applies if curl is replaced by div. Check the box which match the formulas of the vector fields with the corresponding picture I,II,III or IV. Mark also the places, indicating the vanishing or not vanishing of curl and div. In each of the four lines, you should finally have circled three boxes. No justifications are needed.

Vectorfield	I	II	III	IV	curl$(F) = 0$	curl$(F) \neq 0$	div$(F) = 0$	div$(F) \neq 0$
$\vec{F}(x,y) = (0,5)$								
$\vec{F}(x,y) = (y,-x)$								
$\vec{F}(x,y) = (x,y)$								
$\vec{F}(x,y) = (2,x)$								

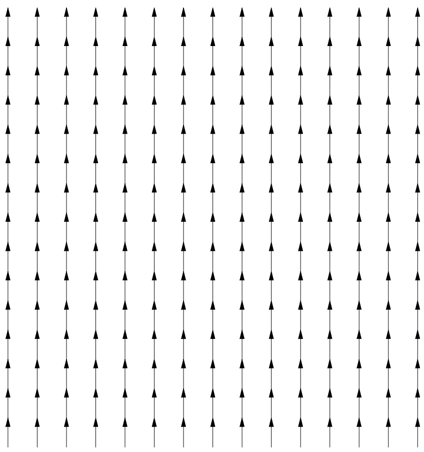
I



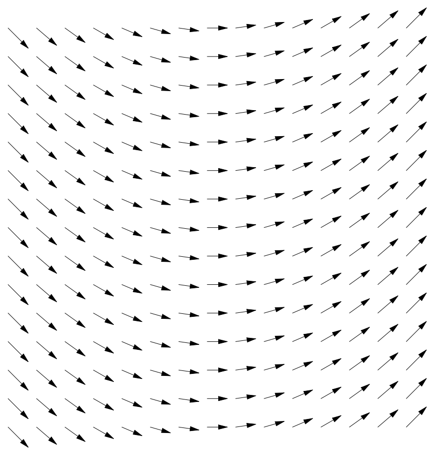
II



III



IV



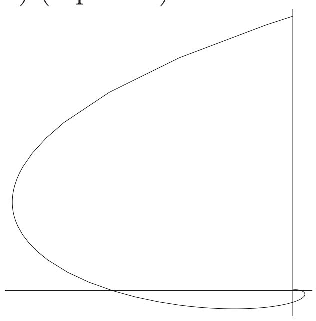
Problem 4) (10 points)

a) Find the scalar projection of the vector $\vec{v} = (3,4,5)$ onto the vector $\vec{w} = (2,2,1)$.

b) Find the equation of a plane which contains the vectors $\langle 1,1,0\rangle$ and $\langle 0,1,1\rangle$ and contains the point $(0,1,0)$.

Problem 5) (10 points)

a) (5 points) Find the surface area of the ellipse cut from the plane $z = 2x + 2y + 1$ by the cylinder $x^2 + y^2 = 1$.
b) (5 points) Find the arc length of the plane curve $\vec{r}(t) = (\sin(t)e^t, \cos(t)e^t)$ for $t \in [0, 2\pi]$.



Problem 6) (10 points)

a) Verify that if $u(x,y)$ and $v(x,y)$ are two functions, then $(uv)_{xx} = u_{xx}v + 2u_x v_x + uv_{xx}$.
b) The identity $\Delta(uv) = (\Delta u)v + u(\Delta v) + 2\nabla u \cdot \nabla v$ holds.
c) Assume u and v satisfy the Laplace equation $\Delta u = u_{xx} + u_{yy} = 0$ and $\nabla u \cdot \nabla v = 0$ then uv satisfies the Laplace equation.

Problem 7) (10 points)

Let $f(x,y,z) = 2x^2 + 3xy + 2y^2 + z^2$ and let R denote the region in space, where $2x^2 + 2y^2 + z^2 \leq 1$. Find the maximum and minimum values of f on the region R and list all points, where the maximum and minimum values are achieved. Distinguish between local extrema in the interior and extrema on the boundary.

Problem 8) (10 points)

Sketch the region of integration of the following iterated integral and then evaluate the integral:

$$\int_0^{\pi} \left(\int_{\sqrt{z}}^{\sqrt{\pi}} \left(\int_0^{x} \sin(xy) dy \right) dx \right) dz \ .$$

Problem 9) (10 points)

Evaluate the line integral

$$\int_C \vec{F} \cdot \vec{dr} \ ,$$

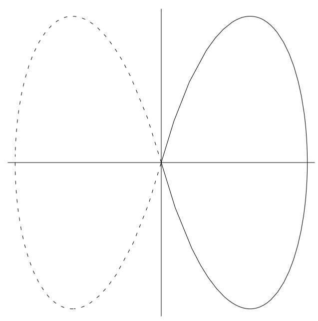
where C is the planar curve $\vec{r}(t) = (t^2, t/\sqrt{t+2}), t \in [0,2]$ and $\vec{F}$ is the vector field $\vec{F}(x,y) = (2xy, x^2 + y)$. Do this in two different ways:

a) by verifying that $\vec{F}$ is conservative and replacing the path with a different path connecting $(0,0)$ with $(4,1)$,

b) by finding a function $f(x,y)$ satisfying $\nabla f = \vec{F}$.

Problem 10) (10 points)

Evaluate the line integral $\int_C \vec{F} \cdot \vec{dr}$, where $\vec{F} = (x + e^x \sin(y), x + e^x \cos(y))$ and C is the right handed loop of the lemniscate described in polar coordinates as $r^2 = \cos(2\theta)$. The lemniscate part on the positive x halfplane is oriented counterclockwise.



Problem 11) (10 points)

Let $f(x,y,z)$ be the distance to the surface $x^4 + 2y^4 + z^4 = 1$. Show that f is a solution of the partial differential equation

$$f_x^2 + f_y^2 + f_z^2 = 1$$

outside the curve.

Hint: here no computations are needed. The shape of the surface pretty much irrelevant. What does the PDE say about the gradient ∇f?

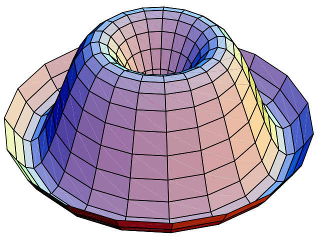
Problem 12) (10 points)

a) Find the line integral $\int_C \vec{F} \cdot d\vec{r}$ of the vector field $\vec{F}(x,y) = (xy, x)$ along the unit circle $C : t \mapsto \vec{r}(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$ by doing the actual line integral.

b) Find the value of the line integral obtained in a) by evaluating a double integral.

Problem 13) (10 points)

Consider the surface given by the graph of the function $z = f(x,y) = \frac{100}{1+x^2+y^2} \sin\left(\frac{\pi}{8}(x^2+y^2)\right)$ in the region $x^2 + y^2 \leq 16$. The surface is pictured to the right.



A magnetic field $\vec{B}$ is given by the curl of a vector potential $\vec{A}$. That is, $\vec{B} = \nabla \times \vec{A} = \mathrm{curl}(\vec{A})$ and $\vec{A}$ is a vector field too. Suppose

$$\vec{A} = \langle z\sin(x^3), x\left(1 - z^2\right), \log(1 + e^{x+y+z})\rangle \ .$$

Compute the flux of the magnetic field through this surface. The surface has an upward pointing normal vector.

Problem 14) (10 points)

Let S be the surface given by the equations $z = x^2 - y^2$, $x^2 + y^2 \leq 4$, with the upward pointing normal. If the vector field $\vec{F}$ is given by the formula $\vec{F}(x,y,z) = \langle -x, y, \sqrt{x^2+y^2}\rangle$, find the flux of $\vec{F}$ through S.