1. a) The tangent plane is the plane through the given point which is orthogonal to the gradient. Here, 

\[ \nabla g = \left( -3y + z + 2, 2y - 3x, x - 1 \right). \]

At \((-1, 0, -1)\), one has \( \nabla g = (1, 3, -2) \). Thus, the tangent plane consists of the points \((x, y, z)\) which obey \( x + 3y - 2z - 1 = 0 \).

b) A normal vector to the plane where \( x = y \) is the vector \((1, -1, 0)\). This is proportional to \( \nabla g \) when \( x = 1 \) and \( z = y + 1 \). Furthermore, a point \((1, y, z) = y + 1\) has \( g = 0 \) only when \( y^2 - 3y + 2 = 0 \), which has \( y = 1 \) or \( y = 2 \). Thus, \( P = (1, 1, 2) \) or \( P = (1, 2, 3) \).

c) \( L(x, y, z) = x + 3y - 2z - 1 \) has the same value as \( g \) at \((-1, 0, -1)\) and the same gradient.

d) Any vector of \( u \) which obeys \( \nabla g \cdot u = 0 \). Thus, \( u = (-3a + 2b, a, b) / \sqrt{10a^2 + 5b^2 - 12ab} \), where both \( a \) and \( b \) are not zero. For instance, \( u = (-3, 1, 0) / \sqrt{10} \).

2. a) The hottest point on the surface is \((\sqrt{3}/2, 0, 1/2)\).
b) The hottest point inside or on the surface is \((\sqrt{3}/10, 0, 1/10)\).
c) The coldest point is \((-\sqrt{3}/2, 0, -1/2)\).

The extreme points on the surface are obtained by solving the Lagrange multiplier equations for those points, where \( x^2 + y^2 + z^2 = 1 \) and \( \nabla T = \lambda \nabla g \), where \( g(x, y, z) = x^2 + y^2 + z^2 \). Here, \( \nabla T = 10(\sqrt{3}/2, 0, 1) - 100(x, y, z) \) and \( \nabla g = (2x, 2y, 2z) \). The extreme point inside is obtained by solving for the points where \( x^2 + y^2 + z^2 < 1 \) and \( \nabla T = 0 \).

3. a) \( r'(0) = i + j \) and \( r''(0) = 2j \).
b) \( s'(0) = -j \) and \( s''(0) = -i \).
c) Write \( \nabla T = a\hat{i} + b\hat{j} \). Then we are told that \( \nabla T \cdot r'(0) = a + b = 3 \) and \( \nabla T \cdot s'(0) = -b = -1 \). Thus, \( b = 1 \) and \( a = 2 \) and \( \nabla T = 2i + j \) at \((1, 0)\).

4. a) The stationary points occur where \( \nabla f = (2xy - 4y, x^2 - 4x + y^2) = 0 \). These are \((2, \pm 2), (0, 0)\) and \((4, 0)\).
b) \((2, 2)\) is a local minimum, \((2, -2)\) is a local maximum, \((0, 0)\) is a saddle, \((4, 0)\) is a saddle. The 2\textsuperscript{nd} derivative test establishes these assertions since the matrix \( f \) of 2\textsuperscript{nd} derivatives has \( \det(f''') > 0 \) and \( \text{trace}(f''') > 0 \) at \((2, 2)\), and \( \det(f''') > 0 \) and \( \text{trace}(f''') < 0 \) at \((2, -2)\) and \( \det(f''') < 0 \) at \((0, 0)\) and \((4, 0)\).
c) The direction of maximum increase is \((1, 1) / \sqrt{2} \). That of maximum decrease is \((-1, -1) / \sqrt{2} \).
d) Any vector of the form \((a, 0)\) with \( a \neq 0 \).

5. The best linear approximation to \( f \) at \((0, 25)\) is \( L(x, y) = 5 + 5x + (y - 25)/10. \) Using \( L \) to estimate \( f \) gives \( 5 + .5 + .03 = 5.53 \).

6. After doing the y integration, one is left with integrating \( x^3(1 - x^2)/2 \) between 0 and 1. The integral is \( 1/24 \).

7. Do the y integral first. (There is no closed form expression for the x integral if that one is done first.) The range for the y integral is from \( y = 0 \) to \( y = x \). The resulting x integral is for the function \( 2xe^{x^2} \) with the range going from \( x = 0 \) to \( x = 1 \). Changing variables to \( u = x^2 \) shows that this is the same as the integral of \( e^u \) from \( u = 0 \) to \( u = 1 \), which is \( e - 1 \).