Some True-False Review Problems

Mark with a true assertion below with a T and a false one with F.

1. If \( u(x, y, z) \) obeys \( u_x + u_y + u_z = 3 \), then \( u \) cannot have a saddle point.

2. The vector \( \mathbf{v} = (2, -2, 1) \) is normal to the level surface of \( f(x, y, z) = 2x^2 + y^2 + z^2 - 7 \) at \((1, 2, 1)\).

3. Some level surfaces of \( f(x, y, z) = \sin((4x + 2y + z)^2) \) are spheres.

4. The minimum value of \( f(x, y) = (x - 2)^2 + (y - 3)^2 \) where \( x^2 + y^2 \leq 1 \) is achieved on the boundary where \( x^2 + y^2 = 1 \).

5. The best linear approximation to \( f(x, y) = 2x^2 + y^3 \) near \((1, 1)\) is \( L(x, y) = 4x + 3y - 4 \).

6. If \( h(x, y) = f(x) \cdot g(y) \) and \( R \) is the region where both \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \), then the integral over \( R \) of \( h \) is the product of the integrals of \( f(x) \) and \( g(y) \), both from 0 to 1.

7. If \( f(x, y) = e^{\cos(2x-y)} (2x - y)^4 \) and \( \mathbf{u} = \frac{1}{\sqrt{5}} (2, 1) \), then the directional derivative of \( f \) in the direction of \( \mathbf{u} \) is zero at all points.

8. For any function \( f(x, y) \), the equality \( \int_0^1 \left( \int_0^{2-2x} f(x, y) \, dy \right) \, dx = \int_0^2 \left( \int_0^{1-y/2} f(x, y) \, dx \right) \, dy \) holds.

9. The function \( (x^2 + y^2)^{1/2} \) has a global minimum at \( x = y = 0 \).

10. If \( f(x, y) = g(x^2 - y^2) \) where \( g \) is any function of one variable, then \( y \, f_x = x \, f_y \).

11. If \( g(x, y) \) is any function of two variables and if \( R \) is the square where both \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \), then \( \int_R g_{xy} \, dA = g(1, 1) + g(0, 0) - g(1, 0) - g(0, 1) \).

12. The only solution to the equation \( u_t = 2u_x \) is \( u(t, x) = 2t + x \).

13. If \( f \) and \( g \) are any two functions of one variable and \( u(t, x) = f(x + 3t) + g(x - 3t) \), then \( u \) satisfies \( u_t = 9u_{xx} \).
**Answers**

1. True. In fact, such a function does not have any stationary points.

2. False. The vectors proportional to $(4, 4, 2)$ are normal at that point.

3. False. All level surfaces are planes of the form $4x + 2y + z = \text{constant}$.

4. True. The minimum is at $\frac{1}{\sqrt{13}}(2, 3)$.

5. True.

6. True.

7. False. The gradient of $f$ is proportional to $(2, -1)$ whose dot product with $\mathbf{u}$ is not zero.

8. True.


10. False. In fact, $y f_x = -x f_y$.

11. True. For a proof, do the $x$-integral first and integrate by parts. Then, do the $y$-integral and integrate by parts.

12. False. Any $u(t, x)$ of the form $h(2t + x)$ satisfies this equation.

13. True.