Answers to Math 21a Hourly 1
(Fall 2001)

1. a) F  b) T  c) T  d) T  e) T  f) F

2. a) T or F depending on your interpretation.  b) F  c) T
d) $2x + y - 3z = 2$. Alternately, you can $\prod = (1, 0, 0) + t \mathbf{a} + s \mathbf{b}$ where $t$ and $s$ are real numbers and $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors that have zero dot product with $\mathbf{n}$.

Note that a)-c) counted 2 points each while d) counted for 4 points.

3. a) $\mathbf{q}'(t) = \frac{1}{\sqrt{2}} (-\sin(t) \mathbf{e}^i + \cos(t) \mathbf{e}^i, \cos(t) \mathbf{e}^i + \sin(t) \mathbf{e}^i)$.
   
b) As $|\mathbf{q}'(t)| = \mathbf{e}^i$, the length of the path is $\int_0^1 \mathbf{e}^i dt = \mathbf{e} - 1$.

4. The answer is $t = (\frac{1}{2} + n) \pi$, where $n$ is any integer.
   Solution 1: Find that $y'(t) = 2 \cos(t) (\sin(t) + 4)$, and so vanishes only where $\cos(t)$ does. In this regard, $x(t) = (8 + \sin(t)) \cos(t)$ and $y(t) = (8 + \sin(t)) \sin(t)$.
   Solution 2: Draw the graph.

5. a) If $\mathbf{Q}=(1,0,0)$, $\mathbf{P}=(0,0,0)$, then $\mathbf{P-Q}$ is in the plane as well as $\mathbf{v}=(4,2,-4)$.
   The vector $\mathbf{n}=(\mathbf{P-Q}) \times \mathbf{v}=(0,4,2)$ is normal to the plane so that $4y+2z=d$ is the equation of the plane. Because $(0,0,0)$ is on the plane, $4y+2z=0$.
b) Use the formula $|\mathbf{v} \times (\mathbf{P-Q})|/|\mathbf{v}|=\sqrt{17}/3$
c) Use the formula $|\mathbf{n} \cdot (\mathbf{P-Q})|/|\mathbf{n}|=3/\sqrt{5}$

6. a) (-10, -2, 500)
b) Coyote hits the ground at $t = 0$. At this point, Coyote’s coordinates are (0, -2, 0) while Road Runner’s are (0, 0, 0). Thus, Coyote is at distance 2 from Road Runner.
c) Coyote’s velocity vector is (1, 0, -100 – 10 t), so equals (1, 0, -100) at $t = 0$ when the ground is hit. His speed is the length of this vector, $(10,001)^{1/2}$.
d) The cosine of the angle is the ratio of the dot product of the velocity vector with the unit vector (0, 0, 1). Thus, the cosine is $-100 (10,001)^{1/2}$. 