40. a) \[ \int_{1}^{5} \left( \int_{3 / 4}^{3 / 4 - v^2 / 25} \frac{\sqrt{1 + 4 (u^2 / 81 + v^2 / 625)}/(1 - u^2 / 9 - v^2 / 25)}{du} \right) dv. \]

b) \[ \int_{1}^{5} \left( \int_{3 / 4}^{3 / 4 - v^2 / 25} \frac{\sqrt{1 + 9 (u^2 / 16 + v^2 / 625)}/(1 - u^2 / 4 - v^2 / 25)}{du} \right) dv. \]

c) \[ \int_{-1}^{1} \left( \int_{-v^2}^{v^2} \frac{\sqrt{1 + 4 u^2 + 16v^6}}{du} \right) dv. \]

41. 10/3. (The area of the surface is \( \pi \) and the integral of \( z \) over the surface is \( 10\pi/3 \).)

42. \( \sin(1) \).

43. 500 \( \pi/3 \).

44. \((x, -y, 0)\).

45. a) \((x^2yz/2, 0, 0)\).

b) \((2z, 3x, y)\).

46. a) \((0, x/\pi)\).

b) \((x, 0, 0)\).

47. a) No such vector field exists because \((x, -2y, xy)\) has divergence -1 and the divergence of a curl is zero.

b) \((xy \cos(yz^2), 0, 0)\).

48. \(5x + 3y = 0\).

49. \(\iint_S x \, g \, dS\). Let \(E = (g, 0, 0)\); here \(\text{div}(E) = f\) while \(E \cdot \mathbf{n} = x \, g\) on the surface of the volume, \(V\), in question. Thus, the divergence theorem implies that \(\iint_S x \, g \, dS = \iiint_V f \, dV\).

50. a) \(2\pi\). This is a direct computation: Parameterize the circle by \(t \rightarrow (\cos(t), \sin(t))\) and then path integral is just the integral of \(dt\) between 0 and \(2\pi\).

b) \(2\pi\). Use Green's theorem for a region with holes and note that the vector field in question has zero 'curl' in the sense that when written as \((P, Q)\), then \(Q_x = P_y = 0\).

c) \(0\). This is also a direct application of Green's theorem.

51. \(A = (-1, 0, 0, 0, 3), B = (-1, -1, 0, 2, 2)\).

52. The mean is \(m = (n_1 \, m_1 + n_2 \, m_2)/(n_1 + n_2)\) and the standard deviation is \(s = ((n_1 - 1) \, s_1^2 + (n_2 - 1) \, s_2^2)^{1/2}/(n_1 + n_2 - 1)^{1/2}\).