Math 21a Hourly 2 Answers

1. a) \( \nabla f = (3, 4) \).
   
   b) The tangent plane is given by \( z - 3x - 4y = -5 \).
   
   c) Using the linear approximation, \( f(1.02, 2.05) = f(1, 2) + \nabla f|_{(1,2)} \cdot (.02, .05) = 6.26 \).

2. The stationary points occur at \((e^{-a}, e^{-b}, e^{-c})/(e^{-a} + e^{-b} + e^{-c})\).

3. 8/3.

4. a) The stationary points are \((0, 1), (0, -1), (1, 1), (-1, 1), (-1, -1)\).
   
   b) The local maximum is \((0, -1)\). The local minima are \((1, 1), (-1, 1)\). The remaining three are saddles.
   
   c) If the level set is tangent to the y axis, then \( \nabla f \) is orthogonal to \((0, 1)\) and so \( f_y = 0 \).
   
   This occurs where \( y = \pm 1 \). Where \( y = 1 \), \( f = x^4 - 2x^2 - 6 \) and so \( f = 2 \) if \( x = \pm 2 \).
   
   Where \( y = -1 \), \( f = x^4 - 2x^2 + 6 \) and so \( f \) is not equal to 2 for any value of \( x \). Thus, the points are \((2, 1)\) and \((-2, 1)\).

5. Change the order of integration to write this integral as \( \int_0^{\pi/2} \left( \int_{\sin(y)}^1 dx \right) dy = \pi/2 - 1 \).

6. Let \( z \) denote height, \( x \) denote length and \( y \) denote width. The you are asked to minimize the function \( f(x, y, z) = 50x + 20 (xz + yz) \) where \( x, y \) and \( z \) are constrained by the requirement \( xy z = 20,000,000 \). The minimum has \( z = 500 \) centimeters and \( x = y = 200 \) centimeters.