Welcome to Math 1b
Calculus, Series, and
Differential Equations

Thomas W. Judson (Course Head)
Spring 2006
About Math 1b
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* Speaking the language of modern mathematics requires fluency in infinite series, integration, and differential equations.
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* We will learn to model practical situations using integrals and differential equations.
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* We will learn how to represent interesting functions using series and find qualitative, numerical, and analytic ways of studying differential equations.
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* Speaking the language of modern mathematics requires fluency in infinite series, integration, and differential equations.
* We will learn to model practical situations using integrals and differential equations.
* We will learn how to represent interesting functions using series and find qualitative, numerical, and analytic ways of studying differential equations.
* We will develop both conceptual understanding and the ability to apply it.
Learning Objectives
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- Integration and Applications of the Definite Integral—
  Techniques of integration, numerical integration,
  areas, volumes, arc length, applications to the natural
  sciences and economics.
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* Integration and Applications of the Definite Integral—Techniques of integration, numerical integration, areas, volumes, arc length, applications to the natural sciences and economics.
* Infinite Sequences and Series—Sequences, series, testing for convergence, power series, and Taylor series.
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* Integration and Applications of the Definite Integral—
  Techniques of integration, numerical integration, areas, volumes, arc length, applications to the natural sciences and economics.

* Infinite Sequences and Series—Sequences, series, testing for convergence, power series, and Taylor series.

* Differential Equations—Modeling, differential equations from a numerical, an analytical, and a geometrical approach, systems of differential equations.
The Textbook

Calculators
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* Any serviceable graphing calculator
Calculators

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* If you plan to purchase a graphing calculator, we recommend the TI-84, TI-86, or TI-89.
Calculators

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* If you plan to purchase a graphing calculator, we recommend at TI-84, TI-86, or TI-89
* Calculators are allowed on homework but not on exams
Techniques of Integration

- \( \int u \, dv = uv - \int v \, du \)
- \( \int \sin^2 \theta \, d\theta \)
- \( \int \frac{1}{x\sqrt{4x^2 - 9}} \, dx \)
- \( \int \frac{5}{(2x + 1)(x - 2)} \, dx \)
Applications of Integration

* How much work is done when pumping out all of the full contained in cylindrical tank?
Sequences and Series

Computing $\pi$

\[
\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots
\]

Representing Functions with Power Series

\[\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\]
Differential Equations

**Predator-Prey Systems**

\[ \frac{dx}{dt} = \alpha x - \beta xy \]
\[ \frac{dy}{dt} = -\gamma y + \delta xy \]
Prerequisites for Math 1b
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* A satisfactory score on the Math Placement Exam
Prerequisites for Math 1b

* A satisfactory score on the Math Placement Exam
* Math 1a
Prerequisites for Math 1b

- A satisfactory score on the Math Placement Exam
- Math 1a
- Math Xa and Xb
To Get a Second Opinion
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* If you still need advice about which math course to take, see a math advisor
To Get a Second Opinion

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* [http://abel.math.harvard.edu/sectioning/index.html](http://abel.math.harvard.edu/sectioning/index.html)
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* [http://abel.math.harvard.edu/sectioning/index.html](http://abel.math.harvard.edu/sectioning/index.html)
* The Online Placement Exam
Sectioning
Sectioning

* Math 1b is taught in small sections
Sectioning

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* You must section by 12:00 PM on Thursday, February 2
Sectioning

- Math 1b is taught in small sections
- You must section by 12:00 PM on Thursday, February 2
- Sectioning directions can be found at http://abel.math.harvard.edu/sectioning/index.html
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Exam Dates
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* Techniques of Integration Exam—Wednesday, February 22 at 6–7 PM in Science B
Exam Dates

- Techniques of Integration Exam—Wednesday, February 22 at 6-7 PM in Science B
- Techniques of Integration Exam Retest—Tuesday, February 28 at 8-9 PM in Science B
Exam Dates

- Techniques of Integration Exam—Wednesday, February 22 at 6–7 PM in Science B
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- Midterm I—Tuesday, March 14 at 7–9 PM in Science B
Exam Dates

* Techniques of Integration Exam—Wednesday, February 22 at 6–7 PM in Science B
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* Midterm I—Tuesday, March 14 at 7–9 PM in Science B
* Midterm II—Tuesday, April 18 at 7–9 PM in Science B
Exam Dates

* Techniques of Integration Exam—Wednesday, February 22 at 6-7 PM in Science B
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* Midterm II—Tuesday, April 18 at 7-9 PM in Science B
* Final Exam—Tuesday, May 23
Homework
Everyone does the same homework
Homework

* Everyone does the same homework

* There are two homework schedules—one for MWF and one for TuTh classes
Homework

- Everyone does the same homework
- There are two homework schedules—one for MWF and one for TuTh classes
- Homework is due at the beginning of class
Homework

- Everyone does the same homework
- There are two homework schedules—one for MWF and one for TuTh classes
- Homework is due at the beginning of class
- We drop your three lowest scores
Pre-Reading Questions and Pre-Class Surveys
Pre-Reading Questions and Pre-Class Surveys

* Pre-reading problems are assigned for each class period
Pre-Reading Questions and Pre-Class Surveys

- Pre-reading problems are assigned for each class period
- Pre-reading problems are marked with an asterisk
Pre-Reading Questions and Pre-Class Surveys

* Pre-reading problems are assigned for each class period
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* Pre-Class Surveys are assigned for each class period (Q&A Tool 2.0)
Pre-Reading Questions and Pre-Class Surveys

* Pre-reading problems are assigned for each class period
* Pre-reading problems are marked with an asterisk
* Pre-Class Surveys are assigned for each class period (Q&A Tool 2.0)
* Pre-Class Surveys are 5% of your homework grade
Take Advantage of the System
Take Advantage of the System

* Each Math 1b section has an undergraduate course assistant
Take Advantage of the System

* Each Math 1b section has an undergraduate course assistant

* CAs hold 90 minute problem sections each week
Take Advantage of the System

* Each Math 1b section has an undergraduate course assistant
* CAs hold 90 minute problem sections each week
* TF Office Hours
Take Advantage of the System

* Each Math 1b section has an undergraduate course assistant
* CAs hold 90 minute problem sections each week
* TF Office Hours
* New location for the MQC
Which of the following are equal to
\[ \int_1^5 \frac{\ln x}{x} \, dx? \]

Please circle all of the correct answers. You do not need to justify your solution.

(a) \[ \sum_{i=1}^{5} \frac{\ln x_i}{x_i} \Delta x, \text{ where } \Delta x = 4/n \text{ and } x_i = 1 + i\Delta x_i. \]

(b) \[ \lim_{n \to \infty} \sum_{i=1}^{5} \frac{\ln x_i}{x_i} \Delta x, \text{ where } \Delta x = 4/n \text{ and } x_i = 1 + i\Delta x_i. \]

(c) \[ \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\ln x_i}{x_i} \Delta x, \text{ where } \Delta x = 4/n \text{ and } x_i = 1 + i\Delta x_i. \]

(d) \((\ln 5)^2/2\)

(e) \[ \frac{1}{2} \left( \frac{\ln 5}{5} \right)^2 - \frac{1}{2}(\ln 1)^2 \]

(f) \[ \frac{1}{5^2} - \frac{1}{1^2} \]

(g) \(\ln(\ln(5)) - \ln(\ln(1))\)

(h) \[ \frac{\ln 1}{1} + \frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} \]
Put the following in *ascending* order (with “=” or “<” signs between each expression. You do not need to justify your solution. [Hint: Think about which expressions are positive, which are negative, and which are zero. A picture may be helpful.]

(a) \( \int_2^6 \ln t \, dt \)

(b) \( \ln 2 + \ln 3 + \ln 4 + \ln 5 \)

(c) \( \ln 3 + \ln 4 + \ln 5 + \ln 6 \)

(d) zero

(e) \( \ln(2/6) \)

(f) \( \lim_{h \to 0} \frac{\ln(2 + h) - \ln 2}{h} \)
Important Addresses

* Thomas W. Judson (Course Head)
  judson@math.harvard.edu

* Course Web Site http://
  www.courses.fas.harvard.edu/~math1b/

* Sectioning Directions http://
  abel.math.harvard.edu/sectioning/index.html