VI.4 The probability \( p \) that a specific player has four aces is \( \binom{48}{9}/\binom{52}{13} \) (choosing 9 cards out of 48 non-aces). The probability of this player not get four aces in \( x \) deals is \((1 - p)^x\); we need \( x \) so that \((1 - p)^x \leq 1/2\). This requires \( x \leq \log(1/2)/\log(1 - p) \approx 262.1\), so 263 deals are necessary.

The probability that some player has four aces is \( 4p \), so the probability of nobody getting four aces in \( x \) deals is \((1 - 4p)^x\). Solving for \((1 - 4p)^x \leq 1/2\) yields \( x \leq \log(1/2)/\log(1 - 4p) \approx 65.3\), so 66 deals are necessary.

VI.5 Note that \( P\{\text{exactly } i \text{ hits on target}\} = \binom{10}{i} \left(\frac{1}{6}\right)^i \left(\frac{4}{6}\right)^{10-i} \). Hence
\[
P\{\text{at least two hits}\} = 1 - P\{\text{no hits}\} - P\{\text{one hit}\} = 1 - (4/5)^{10} - 10(4/5)^9 \approx 0.624.
\]

VI.8 First we calculate the probability that the birthdays fall in two specified months (say \( A \) and \( B \)) and at least once in \( A \) and in \( B \). Each birthday has a \( 2/12 = 1/6 \) chance of being in \( A \) or \( B \), so the probability that all birthdays are in \( A \) and \( B \) is \((1/6)^6\). However, this includes the cases that all birthdays are in one of the two months, so we need to subtract off \( 2(1/12)^6 \). Hence the probability of the birthdays falling in \( A \) and \( B \) and at least one in each is \((1/6)^6 - 2(1/12)^6\).

Now to solve the actual problem, we can just multiply \((1/6)^6 - 2(1/12)^6\) by the number of ways to choose two months. Notice that there is no overcounting. Hence the final answer is \( \binom{12}{2} \left((1/6)^6 - 2(1/12)^6\right) \).