TAYLOR FORMULA. Because \( \frac{df(z(t))}{dt} = f(x + r \cos(t) + iv + r \sin(t)) = f(w)(r \cos(t) + iv \sin(t)) = f(w)(z - w) \), this can be rewritten as \( \int_{jw}^{zw} f(w)(z-w) \, dz = f(z) \). This is the Cauchy integral formula.

Since we can differentiate the left hand side arbitrarily often with respect to \( z \), this proves that an analytic function is arbitrarily often differentiable and \( f(w)/(z-w) \) has the \( n \)th derivative \( \frac{f^{(n)}(z)}{n!} \), we get
\[
f(w) = \sum_{n=0}^\infty \frac{f^{(n)}(z) (w-z)^n}{n!}
\]
which is the familiar Taylor formula if \( f \) is real.

CAUCHY THEOREM. The Cauchy Riemann equations also prove the Cauchy formula.
If \( C \) is a closed curve in simply connected region \( U \) in which \( f \) is analytic, then
\[
\int_C f(z) \, dz = \int f(z(0)) z'(t) \, dt = 0
\]
because the latter is the line integral of \( F(x,y) = -(\bar{v}(x,y), u(x,y)) \) and Greens theorem in multi-variable calculus shows that curl(\( F \)) = curl(\( -u \)) = (\( u_y, -v_x \)) = 0. In other words, the vector-field \( F(x,y) = -(\bar{v}(x+iy), u(x+iy)) \) is conservative.

FIXED POINTS. Because the eigenvalues of the rotation dilation \( A \) come in complex conjugate pairs, the fixed points or periodic points can not be hyperbolic. Fixed points are either stable sinks, or unstable sources elliptic, conjugated to a rotation. For example, the fixed points of \( f(z) = z^2 + c \) are \( (1 \pm \sqrt{1 - 4c})/2 \) and the linearization at those points is \( df(z) = (1 \pm \sqrt{1 - 4c})z \).

TOPOLOGY. Here are some topological notions occurring in complex dynamics.
OPEN. A set \( U \) in the plane is called open if for every point \( z \), there exists \( r > 0 \) such that \( B_r(z) = \{ w | |w-z| < r \} \) is contained in \( U \). One assumes the empty set to be open. The entire plane is open too.
CLOSED. A set \( U \) in the plane is closed, if the complement of \( U \) is open. The entries plane is closed.
INTERIOR. The interior of a set \( U \) is the subset of all points \( z \) in \( U \) for which there exists \( r > 0 \) such that \( B_r(z) \subseteq U \). If \( U \) is open, then \( U \) is equal to its interior.
BOUNDARY. The boundary of a set \( U \) is the closure of \( U \) minus the interior of \( U \). The boundary of a closed set without interior is the set itself.
SIMILY CONNECTED. A set \( A \) is simply connected, if every closed curve contained in \( A \) can be deformed to a point within \( A \). A simply connected subset of the plane has no "holes".
CONNECTED. A set \( A \) is called connected if one can not find two disjoint open sets \( U, V \) such that \( A \cap \overline{U} \neq \emptyset, A \cap \overline{V} \neq \emptyset \).

A set \( A \) is connected if and only if the complement is simply connected.

To verify that the complement of \( M \) is simply connected, one finds a smooth bijection of the complement of the unit disc with the complement of \( M \). The bijection is given by \( \Theta(z) = \lim_{n \to \infty} (f^n(z))^{1/2^n} \). The Mandelbrot set \( M \) is connected as well as simply connected. The Julia sets \( J_c \) are connected if \( c \) is in \( M \).
COMPACT. A subset of the complex plane is called compact if it is closed and bounded. A sequence in a compact set always has accumulation points. The Mandelbrot set as well as the Julia sets are examples of compact sets.
PERFECT SETS. A subset \( J \) in the complex plane is perfect if it is closed and every point \( z \) in \( J \) is an accumulation point for \( z \) in \( J \). Perfect sets contain no isolated points.
NOWHERE DENSE. A subset \( J \) in the complex plane is nowhere dense if the interior of its closure is empty. A Julia set \( J_c \) is nowhere dense if \( c \) is outside the Mandelbrot set.
CANTOR SET. A perfect nowhere dense set is also called a Cantor set. An example is the Cantor middle set. A Julia set \( J_c \) is a Cantor set if \( c \) is outside the Mandelbrot set.