Abstract:
Whether the 3D incompressible Euler equation can develop a finite time singularity from smooth initial data has been an outstanding open problem. Here we review some existing computational and theoretical work on possible finite blow-up of the 3D Euler equation. We show that there is a sharp relationship between the geometric properties of the vortex filament and the maximum vortex stretching. By exploring this local geometric property of the vorticity field, we have obtained a global existence of the 3D incompressible Euler equations provided that the unit vorticity vector and the velocity field have certain mild regularity property in a very localized region containing the maximum vorticity. Further, we perform large scale computations of the 3D Euler equations to re-examine the alleged finite-time blowup of the two antiparallel vortex tubes, which has been considered as the most promising candidate for a finite-time blowup of the 3D Euler equations. Our numerical studies indicate that the maximum vorticity does not grow faster than double exponential in time. The velocity field and the enstrophy remain bounded throughout the computations. The vortex lines near the region of the maximum vorticity are relatively straight. This local geometric regularity of vortex lines seems to be responsible for the dynamic depletion of vortex stretching.